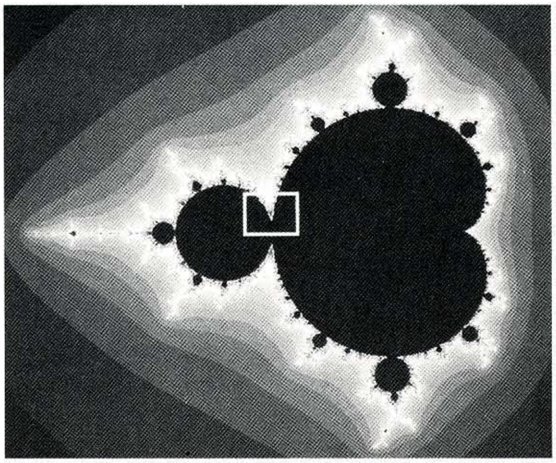
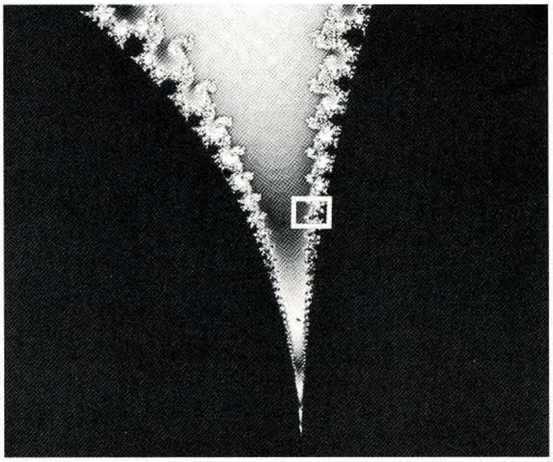
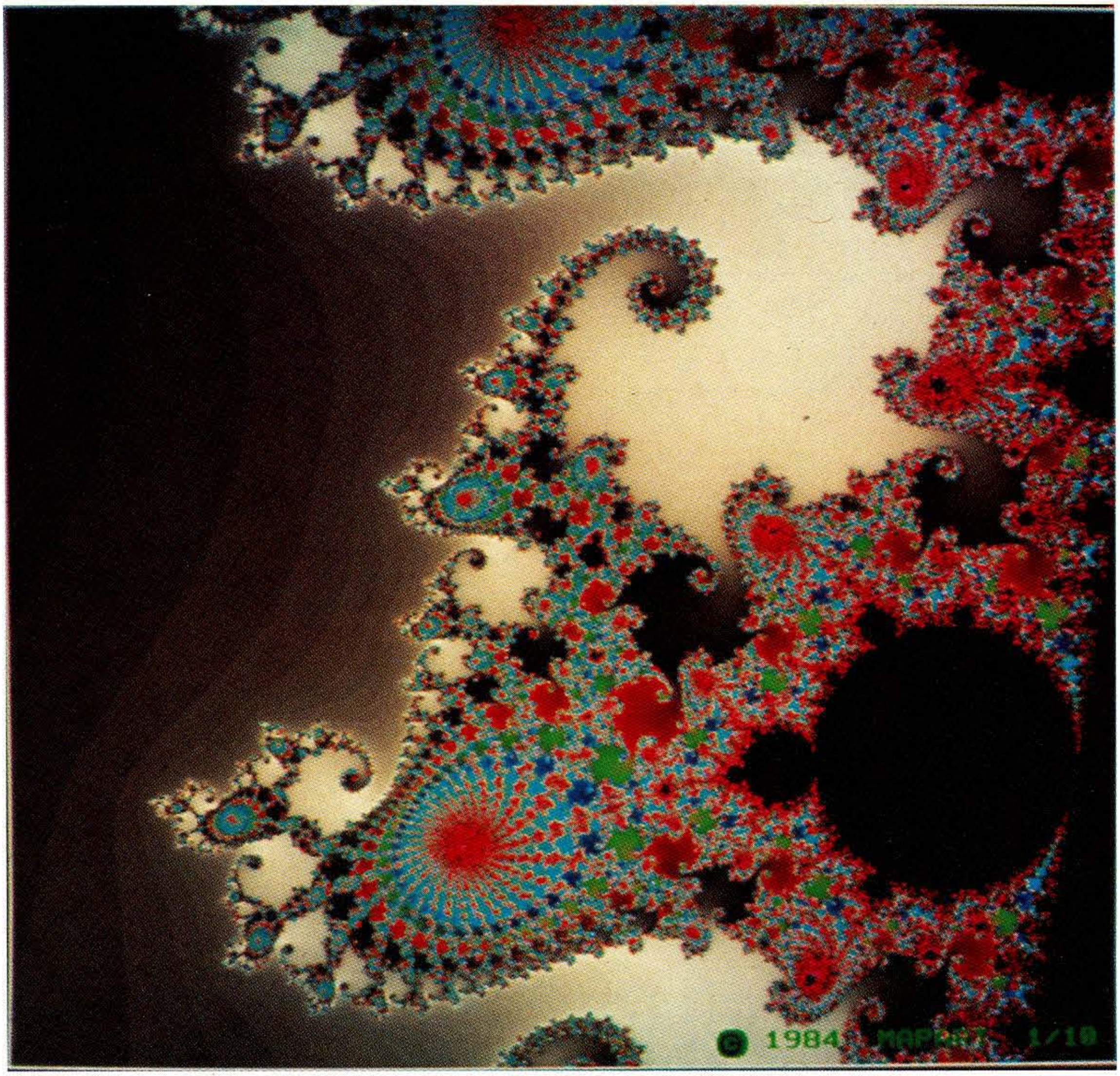






Fractal illustrations pp.2-8 © H.-O. Peitgen, P.H. Richter, *The Beauty of Fractals*. Springer-Verlag, 1986.

Simplicity and Complexity in the Description of Nature

On October 1, 1987, the day this talk was delivered to The Caltech Associates, a 5.9 earthquake rattled Pasadena.

by Murray Gell-Mann

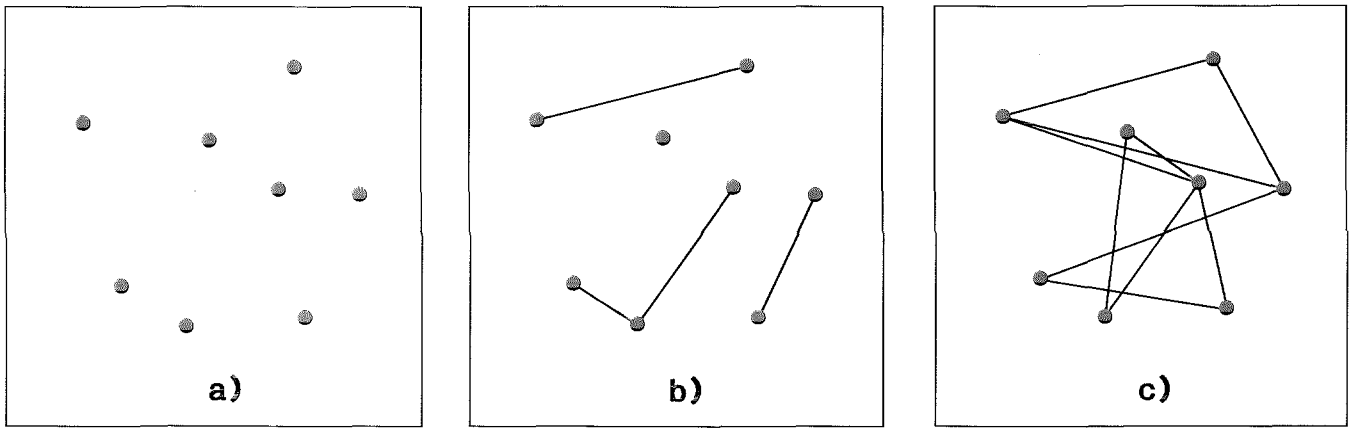
IN PRESENTING THE PICTURE of a fractal, I had hoped to show many of you something new. But in one of my rare moments of watching the idiot box, while riding my exercise bike, I noticed the same picture in an advertisement for IBM boasting of the achievements of their man, Caltech alumnus Benoit Mandelbrot.

If you look carefully at the fractal on the opposite page, you can notice its remarkable self-similarity—that is, the gross structure is composed of structures of the same kind in minature. On the following pages you can see that those smaller structures are made up of still smaller similar structures and so on further down in scale.

Is this fractal a simple system or a complex one? What do the concepts of simplicity and complexity mean, and, in particular, what do they mean in terms of scientific theory? In the description of nature, does deep simplicity always underlie apparent surface complexity? To what extent is the so-called reduction of each level of scientific description to a more basic level possible? When it is possible, to what extent is it a good strategy to pursue?

In the science of ecology, a debate has been going on for decades over whether complex ecosystems like tropical forests are more robust than comparatively simple ones such as the forest of oaks and conifers near the tops of the San Gabriel Mountains—robust, that is, with respect to major perturbations from climate change or fire or other environmental alterations wrought by nature or human activity. Currently those ecological scientists seem to be winning who claim that, up to a point, the simpler ecosystem is more robust. But what do they mean by simple and complex? To arrive at a definition of complexity for forests, they might count the number of species of trees (less than a dozen near the tops of the San Gabriels compared to several hundred in a tropical forest); they might count the number of species of birds, mammals, or insects. (Just imagine the number of kinds of insects in a tropical forest, and note that the estimated number has recently been revised sharply upward as a result of new experiments in which all the insects in a tree are killed and identified.) The ecologists might also count the interactions among the organisms: predator-prey, parasite-host, pollinator-pollinated, and so on. Down to what level of detail would they count? Would they look at microorganisms, even viruses? Would they look at very subtle interactions as well as the obvious ones? Clearly, to define complexity you have to specify the level of detail that you are going to talk about and ignore everything else—to do what we call in physics "coarse-graining."

For example, let us take a parallel-processing computer network such as that being developed by Geoffrey Fox, professor of theoretical physics, Charles Seitz, professor of computer science, and others. Because it is made up of individual computers linked together, they may ask what is a simple and what is a complex pattern of hooking up the constituent computers. We can "coarse-



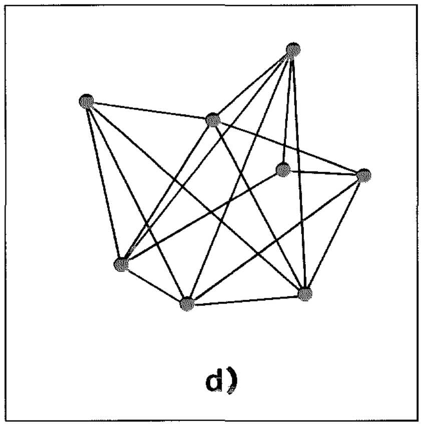
grain" by ignoring any directionality in the links and any differences among them; we can also ignore the identities of the individual computers and their geometric arrangement. Then we get a mathematical system that we can represent simply as an unlabeled bunch of dots, with the positions of the dots not significant. What are simpler ways of connecting, say, eight dots and what are more complex ways? (Actually, complexity is important only when there are lots of individual units, but let's just imagine that eight is a lot.) First we have a pattern a) with nothing connected; that's obviously very simple. We can connect a few dots with one another and still get a fairly simple pattern b). Then we can add more and more links (c, d, and e) until finally we have a pattern f) where all possible connections are made: every point is connected to every other point. An interesting question to ask is whether the pattern with all the dots connected is more complicated than the pattern with no connections. They are similar in that they are opposites of each other. Likewise, b) and e) are opposites and c) and d) are opposites in the sense of interchanging linked and unlinked pairs of dots: two dots that are linked in c) are unlinked in d) and vice versa, etc.

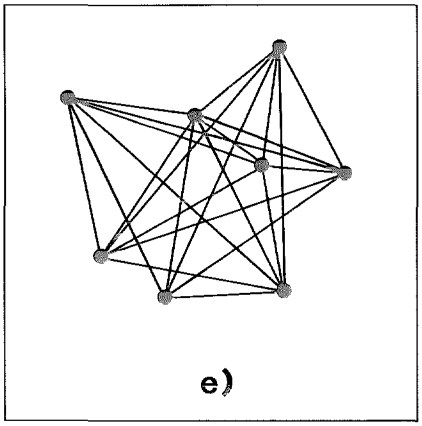
We have discussed examples of what might be simple or complex, but we still don't have a definition. We could try to use as a definition the length of the message that describes the system under discussion. If it's described by a very long message, then, roughly speaking, it would be complex; if it's described by a very short message, it would be simple. But we don't want the message to be what's called an ostensive definition, that is, just pointing to something; it has to be a real description, suitable for distant correspondents. Likewise, we don't want to be able to simplify the message artificially just by calling the thing we are talking about by some pet name. We can call the most complicated thing in the world "Sam"—a very short message indeed. So if we want to define in a serious way the complexity of something, we have to agree beforehand on some grammar and vocabulary for its description. Then it's a question of whether in terms of this particular grammar and vocabulary the message describing the thing is long or short. Finally, the definition shouldn't depend on how clever we are at writing a short message; thus we should use the length of the shortest message that will describe the system for distant correspondents using given vocabulary and grammar.

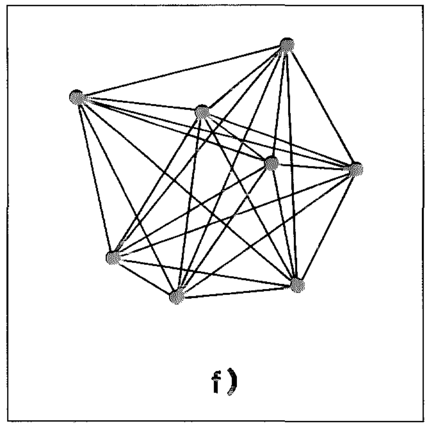
About 20 years ago some mathematicians, including Gregory Chaitin in the U.S. and A.N. Kolmogorov in the Soviet Union, came up with such a definition for ideal complexity. They gave it a slightly more mathematical form and used it in a mathematical context, but it amounts to what I have described above—the minimum length of a message describing a system up to a given level of detail to a distant observer using a given grammar and vocabulary. In the language of computer science, one can speak of the shortest computer program that will cause a particular type of general-purpose computer to print out the description and then stop.

Now this ideal complexity is by no means the only notion of complexity that is needed, but it is a useful one, even though it has some strange properties, as we shall see.

Suppose we consider a message reduced to a string of digits. Sometimes that message can be compressed, that is, replaced with a shorter one with the same meaning. (That's why it is the minimum length of the message that defines the complexity.) Suppose the message is 575757575757575757575757575757 575757575757. We can shorten this message







to "57 repeated 20 times." If the message is "3.141592 . . ." to 250 places, which we recognize as the first 250 digits of π, then we can shorten the message by calling it "π to 250 digits." Thus, when we talk about the minimum length, we mean that all possible compressions of the message have been found and used.

Actually, it can be proved that there is no finite procedure for finding all the compressions. You can never be sure that you've found all the different tricks for shortening a message. Hidden in some math book that you didn't know about, or that hasn't been written yet, or that never will be written, there might be a theorem that would let you compress the message further. Thus the definition has a peculiar flaw in it. Although you can sometimes show that a thing is simple by demonstrating that it can be described by a short message, you can never show for sure that another thing is complex and requires a long message, because an undiscovered way of shortening that message may still exist.

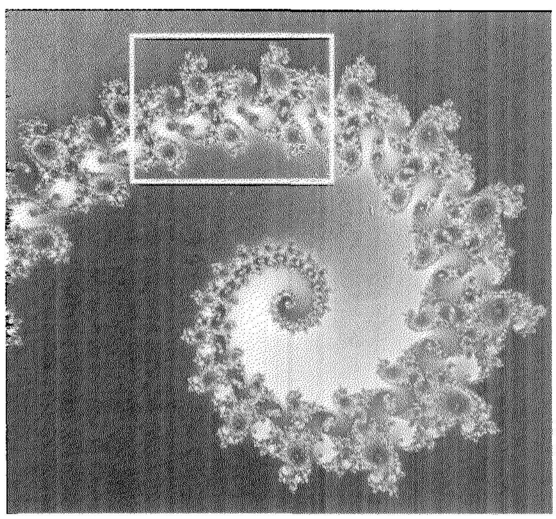
Mathematicians have shown that most long strings of digits have the property of being nearly incompressible (those are called random strings), but we will never know which ones.

We can now go back to the different ways of linking computers, or else dots in a diagram. We see that, from the point of view of ideal complexity, the opposites, such as "all links" and "no links," are about equal in complexity because the shortest messages for the opposites can just have "link" and "no link" interchanged. In the limit of a large number of dots, there will be no difference in complexity between opposites.

Now let us return to our fractal, and discuss how the picture is generated. Suppose that horizontal distance on the plane is measured by x and vertical distance by y, so that every point on the plane is described by the pair of numbers (x,y). We then consider the transformation

$$x \rightarrow x + x^2 - y^2, \qquad y \rightarrow y + 2xy$$

that takes each point into another point. We apply this transformation over and over to each point of the plane. If a given point keeps moving further and further from (0,0) under this procedure, without limit, then it is assigned a color, with the color depending on how fast it moves. If a point does not keep getting further and further from (0,0) without limit, then it appears as black. Since all the information you need to generate the fractal is this simple rule, the fractal is not complex at all from the point of view of ideal complexity. But ideal complexity does not take into account the time, the work, or the expense of generating the picture from the formula—or the difficulty of figuring out how to go back from the fractal to the simple rule that underlies it. (Clearly, other definitions of complexity need to be used from time to time.)



One way of writing a message is to express a system in terms of the sum of its parts. Suppose you try to describe a human body by looking at all the cells separately, then listing the properties of the cells and the way they are arranged in the body, and trying to identify the complexity of a human being with the sum of the complexities of all the cells plus the complexity of the arrangement. You end up with a value that is much too large because the cells are all related to one another. They have the same genes and in many cases a lot of the same chemical properties. They're *organized*, and in fact organization can be defined as the sum of the complexities of the parts and the complexity of the arrangement minus the complexity of the whole. Understanding the organization produces an enormous compression of the message describing the body.

The best way to compress an account of large numbers of facts in nature is to find a correct scientific theory, which we may regard as a way of writing down in concise form a rule that describes all the cases of a phenomenon that can be observed in nature. Stephen Wolfram (another Caltech alumnus) has emphasized this point. A scientific theory thus compresses a huge number of relationships among data into some very short statement. Of course, you need to study for a while to learn how to read that statement.

For example, my father, as a layman, struggled to understand Einstein's general theory of relativity. On one occasion he said, "You know, Einstein's equation in free space is awfully simple. All it says is that $R_{\mu\nu} = 0$, but I have to understand better what $R_{\mu\nu}$ is."

The point is that the lengths of the texts you have to study are finite, and the number of facts described by a successful scientific law is indefinitely large. Thus the complexity of what you have to learn in order to be able to read the statement of the law is not really very great compared to the *apparent* complexity of the data that are being summarized by that law. That apparent complexity is partly removed when the law is found.

Let us take Maxwell's equations for the classical electromagnetic field as an example. When they were discovered, more than a century ago, it became possible to calculate, in principle, the electric and magnetic fields in a volume of space if the conditions on the boundary of that volume were known. Thus the apparent complexity of the fields throughout the space was reduced to the lesser complexity of the boundary conditions. That is typical of what happens when a correct scientific theory makes its appearance. Great quantities of data are explained, but they are explained in terms of the particular circumstances under which the theory is applied—those circumstances must still be specified.

The various laws of nature can be classified according to the level at which you are studying nature. Are you studying it at the most basic level of the fundamental laws of physics? Are you studying it at the level of the rest of physics and chemistry, or the level of some branch of biology, or the level of psychology, or the level of social science, or what? We may recall that in the 19th century Auguste Comte established an order of scientific subjects: mathematics, then physics, then chemistry, then physiology, and so forth; I am told that until quite recently the faculty of sciences at a French university would discuss business in that order. (The concerns of the biologists must have been somewhat neglected as a result.)

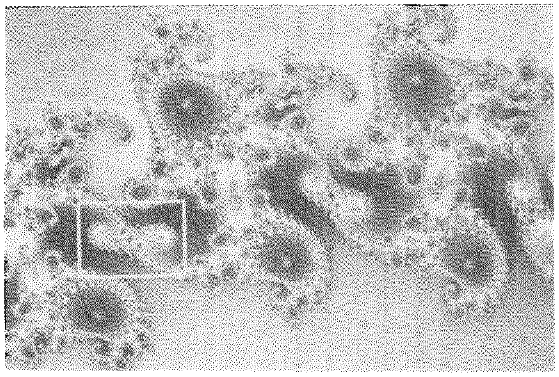
I've spent most of my career working on the most basic level, that of the fundamental laws of physics. Those have a special simplicity, as in Einstein's theory, even though, as we remarked, it takes a little while to learn what the equations mean. We seek two basic principles that underlie all of physics and chemistry. One of those is the unified theory of all the elementary particles (the constituents of all matter in the universe) and of all the forces among them. For the first time in history a likely candidate for such a theory has actually emerged—superstring theory, invented by John Schwarz, professor of theoretical physics here at Caltech, and his associates. It may even be correct.

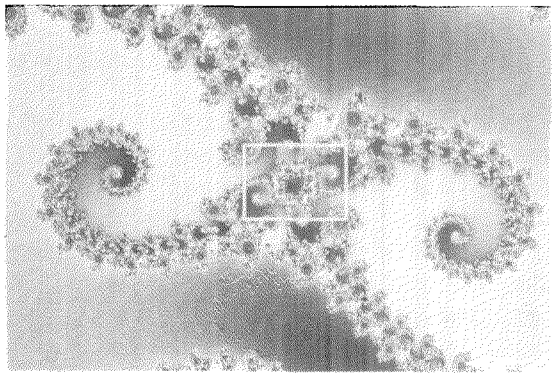
Let us suppose that it is correct, that we have the fundamental theory of the elementary particles and their interactions; what else is there to describing nature at the most basic level? The other principle we need to know is a kind of boundary condition in time, the initial condition of the universe, the character of what is sometimes called the "big bang" that took place some 10 to 15 billion years ago. Is there a simple formula for it? If we believe that the fundamental law of the elementary particles might be described by some relatively simple equation like that of superstring theory, why not go further and conjecture that the initial condition of the universe might also be described in a simple way? A number of guesses at such a description have actually been made, starting with the one by Stephen Hawking of Cambridge University and James Hartle, a Caltech alumnus now a professor at UC Santa Barbara, in their classic paper "The Wave Function of the Universe," which gave a strong impetus to the field of quantum cosmology. Actually it is the simplicity of the early universe that is responsible for the "second law of thermodynamics" that describes the tendency of the entropy of the universe to increase with time. In layman's language, we are talking about the arrow of time that allows us to recognize whether a film of a macroscopic event, such as today's earthquake, is being shown forward or backward—if we see piles of bricks on the ground assembling into chimneys and perching on the tops of buildings, we know time is being made to run backwards.

Now suppose we know both of these fundamental principles, the theory of the elementary particles and the condition of the early universe. Then we have a complete formula that accounts for all the laws of physics and chemistry. Would that tell us in principle about the behavior of everything in the universe? No, it would not, because the theory is quantum-mechanical, and quantum mechanics gives only a formula for probabilities. Much is still up to chance. There are very many throws of the quantum dice in addition to these fundamental laws. And those unpredictable quantum fluctuations are responsible for many of the details of the particular universe that we experience. Quantum mechanics describes many possible universes, but we are interested in the details of *this one*, and a lot of those details depend on the throws of the dice and are not predictable from the formula, except probabilistically.

Even in the approximation of deterministic classical physics, there is the widespread phenomenon known as "chaos" (which is, by the way, connected in interesting ways with fractals). In a "chaotic" situation, the outcome is infinitely sensitive to the initial conditions, and thus, even in the deterministic classical approximation, prediction of details becomes practically impossible. The fundamental indeterminacy of quantum mechanics compounds the situation. Believe it or not, there was a recent editorial in the *Los Angeles Times* on this subject.

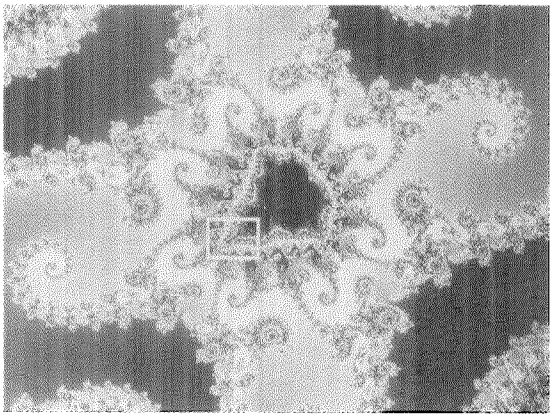
When we look beyond physics and chemistry to what we might call the environmental sciences (astronomy, geology and planetary science, biology, anthropology, human history, and so on), we are dealing with many kinds of detailed events that depend to some extent on inherently unpredictable fluctuations. Much of the information in those details cannot be determined from the fundamental underlying laws. There are patterns and correlations that can be so derived, in principle, but the rest of the information, idiosyncratic for this particular universe, is random and incompressible.

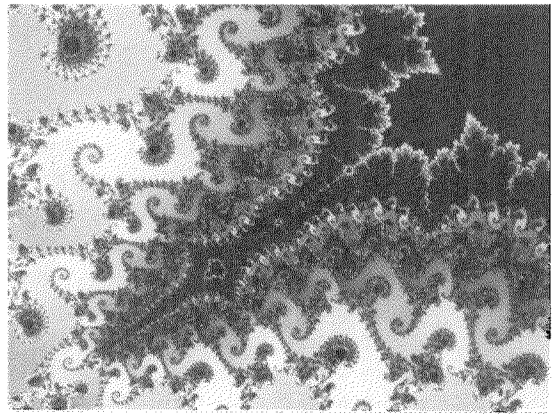




While the statistical distribution of galactic shapes and sizes may be derivable from elementary particle physics and quantum cosmology, the details of the structure of any particular galaxy, such as our own, must depend on particular chance events. Likewise, the detailed characteristics of the solar system are inherently unpredictable, and so are the details of the history of life on Earth. The specific events of human history, including the existence of particular individuals, also depend to a great extent on chance.

Typically, an object of study in the environmental sciences is a complex system, which, however, being organized, is less complex than the sum of its parts, and may be much less complex than it appears to be. Most scientists think that a certain minimum true complexity is needed in order to have life, with its characteristic features of reproduction, variation, and selection. Even more complexity is no doubt required for intelli-





gence, such as we human beings are alleged to possess.

Life may or may not be required by the fundamental principles of physical science to utilize DNA, with the same four nucleotides with which we are familiar; perhaps elsewhere in the universe there are other kinds of life. But even if all life in the universe has the same basic structure, surely the details of particular species that have evolved on the Earth, including our own, are idiosyncratic. In fact, in biological evolution there is an interesting interplay of *fundamental requirements*, *pure accidents* (probably including, for example, the choice of right-handed molecules over left-handed ones), and *survival of characteristics that are adaptive.* The same is true of many other evolutionary processes.

Today, the whole subject of complex adaptive systems, systems that exhibit random variation and selection resulting in learning or evolution, has become extremely exciting. It involves interdisciplinary research spanning a vast number of traditional fields, such as evolutionary biology; psychology and psychiatry, as well as the more fundamental level of neurobiology; linguistics; and many of the social sciences.

Computer science also comes into play, for example where it involves "genetic algorithms," as in the work of John Holland of the University of Michigan, who trains a computer to generate entirely new strategies for solving problems, strategies that no human being has ever developed. By introducing random "mutations" into his computer programs and arranging for the promotion of those parts of the programs that help to achieve a better strategy and the elimination of those parts of programs that get in the way, Holland causes strategies to be evolved in the computer much as life evolves on Earth.

Whereas Holland makes use of existing computers, there are other researchers who are trying to design new types of computers that are intrinsically adaptive in their mode of operation. Here at Caltech, for example, the program called "Computation and Neural Systems" emphasizes computers based on so-called "neural nets" and possible analogies with situations encountered in neurobiology.

The study of adaptive complex systems embraces these efforts in computer science and in neurobiology along with theoretical work, linked to experiment and observations, on such subjects as biological evolution, prebiotic chemical evolution, the operation of the immune system, learning and thinking in the higher animals including humans, and the evolution of human language. Most of these areas of research are now largely or wholly lacking at Caltech, but it is important to include them, because of the ideas and insights that each subject can contribute to the others. Remarkable parallels are starting to turn up in the search for general principles that govern adaptive complex systems.

I mentioned earlier that at every level there are characteristic scientific laws not only at the most fundamental level of elementary particle physics and cosmology, but in the rest of physics and chemistry, astronomy and planetary science, biology, psychology, and the social sciences. Is it possible in principle, and is it wise in practice to try to reduce each level of scientific description to some lower level? Most of us are of course reductionists in the sense that we don't believe that there are mysterious forces explaining chemistry that have nothing to do with physics; or mysterious "vital forces" that explain biology but don't depend on chemistry and physics; or mysterious mental processes responsible for psychology that are not biological, physical, or chemical in character. Nevertheless, we may still ask: Is a full reduction really possible, and as a strategy is it wise to rely on the reduction of one level of science to what seem to be more basic levels?

My own answer is no—for three reasons. First, one of the major activities of science is to build bridges between one level and the next—between the mind and the brain, for example, or between biology and chemistry, or between chemistry and fundamental physics, and so forth. Usually these bridges take a long time to build, and while we're building them, we still need to know about the subject that lies at the higher level of complexity. For instance, we can't wait for the bridge to be completed between geology on the one hand and chemistry and physics on the other, in order to learn about the behavior of the earth. We want seismologists to proceed as rapidly as possible in their work of explaining today's earthquake and not to have to wait until they can derive earthquakes from superstrings.

Second, when we elucidate the patterns that appear at each level of organization, we find that neat and useful laws emerge. Principles of psychology are found long before they can be explained by neurobiology; principles of anthropology are found long before they can be explained by psychology, let alone fundamental biology; and so forth. Furthermore, in building a bridge to the more basic levels of description, it's much easier to relate the laws of the higher level of organization, rather than a mass of raw data, to the laws at the lower level.

Third, there are fundamental limitations to the amount of reduction that can be carried out, even in principle, because of the indeterminacies—particularly the indeterminacy of quantum mechanics.

At each level of description, then, there are many important features of the world around us that are fundamentally unpredictable from the basic laws of physics but depend on the accidents of this particular universe. There are others that for practical reasons are difficult to derive from the laws at lower levels of organization. But there are patterns at each level of description that give the appropriate laws for that level, and I am suggesting that it is among those laws that one tends to find opportunities for practical reduction to more basic levels, with deep simplicity explaining away a great deal of the surface complexity.

There are, of course, many other patterns that can be reduced in principle but not in practice, at least in any reasonable time. But what of the random features that are impossible to reduce? In many cases they are of great scientific interest. For example, suppose it should turn out that life is possible without DNA chains made of the familiar four nucleotides. It is nevertheless very important for us on Earth that life does have that character here—even if it is a local law based on a local accident. Still more striking examples may occur in elementary particle physics, where it may turn out, even in parameter-free superstring theory, that there are various equally valid solutions to the equation, one of which is chosen by our universe. There would then be parameters after all with particular values in our universe. We would be dealing with "local" conditions that prevail throughout the whole universe, and elementary particle physics would, to some degree at least, join the environmental sciences.

Other random features, also of great scientific interest, may have the character of natural history rather than that of analytic science.

In still other cases, there will be random features of lesser interest to science as such, but nevertheless important in other ways, as the fascinating material that gives individuality to the different parts of the world around us— the details in the shape of a cloud, in the individual motions of the birds in a flock, in the appearance of the crystals of various minerals in a particular rock. Those individual details may not appear significantly in scientific laws at any level, but they give richness to our experience of the world, largely through the other, non-scientific modes of apprehending the universe, such as the artistic and aesthetic modes.

No matter how we try to describe the universe, through scientific research, through artistic creation, or through appreciation of its beauties, it exhibits a wonderful interplay of simplicity and complexity. □