LETTERS

Pasadena

EDITOR:

I have compared the original and the published versions of L. E. Fraenkel's review of Saul Kaplun's and J. D. Cole's books (*E&S*—October 1968). The severe editing of the first half of the review has, in my opinion, not only led to omission of important material, but has in various ways distorted the meaning of Fraenkel's review. I hope it will be possible for you to print the original version of the first half of the review.

P. A. LAGERSTROM

For the record, Dr. Fraenkel's original review of Fluid Mechanics and Singular Perturbations *read:*

The theory describing the flow past bodies of a viscous fluid is a notoriously difficult one; the governing equations (Navier-Stokes) can be solved exactly and explicitly for only a few highly artificial cases, and approximation schemes invariably meet difficulties demanding and stimulating increasingly subtle mathematical techniques. In the years 1954-57 there appeared from Caltech four great papers: "The Role of Coordinate Systems in Boundary-layer Theory," S. Kaplun (1954); "Examples Illustrating Expansion Procedures for the Navier-Stokes Equations," P. A. Lagerstrom and J. D. Cole (1955); "Asymptotic Expansions of Navier-Stokes Solutions for Small Reynolds Numbers," S. Kaplun and P. A. Lagerstrom (1957); and "Low Reynolds Number Flow Past a Circular Cylinder," S. Kaplun (1957). These papers not only gave a fresh and clearer view of the various approximation schemes for very viscous and for nearly inviscid flow but also systematized and extended significantly the mathematical technique implicit in Prandtl's boundary-layer theory. Indeed, these papers did much to provide the applied mathematician with new tools that have since then been applied to an astonishing variety of problems. These tools now form part of the subject known as "singular perturbations," a phrase used to describe methods for solving boundary- or initial-value problems which involve a small parameter in such a way that the "obvious" expansion in terms of that parameter fails to approximate the desired solution throughout the physical domain. Saul Kaplun, to whom Lagerstrom and Cole attribute the principal ideas in this work, died in 1964 at the age of 39.

Fluid Mechanics and Singular Perturbations is in two parts. The first contains Kaplun's three published papers and preliminary drafts of intended papers on his approach to singular perturbations and on the lift at low Reynolds number of two-dimensional bodies. The second part contains similar drafts of some of his extensive work on the problem of flow separation. The editor's work consisted of selection, of minor changes with respect to notation and errors of transcription, and of the provision of a most helpful commentary.

The first paper—concerning "optimal co-ordinates," in terms of which the boundary-layer solution contains the entire outer flow not merely to first but to second order—makes an elegant and satisfying beginning. The remainder of Part I shows exceptional depth. The problem of very viscous flow—that is, flow at low Reynolds number—past a circular cylinder seemed anomalous from 1851, when Stokes found that his approximate equations had no appropriate solution, until 1910, when Oseen resolved the difficulty in a manner very satisfactory for fluid mechanics but of an essentially *ad hoc* character. (The application of Oseen's method to the circular cylinder is actually due to Lamb.) Kaplun's approach was systematic; put both the Stokes and the Oseen equations into their proper setting; led to a scheme which could, in principle, be continued to arbitrary order; and gave a new view of the relationship, in delicate problems of this kind, between various expansions of the desired solution. Previously, it had not been suspected that an extension of Prandtl's ideas for flow at large Reynolds number might remedy the difficulties at the other end of the range.

Part II of the book, on the behavior of solutions of the boundary-layer equations near a point of zero skin-friction, is strictly for specialists—and ardent specialists at that. This material is not in the final form that Kaplun hoped to produce, and a number of the results were also found by Stewartson in 1958. Nevertheless, there is no doubt that Kaplun's thorough analysis (first of the Von Mises form of the equations and then of the transformation to the physical plane), with its careful interpretation of the basic meaning of each step, adds substantially to what is known of this difficult aspect of boundary-layer theory.