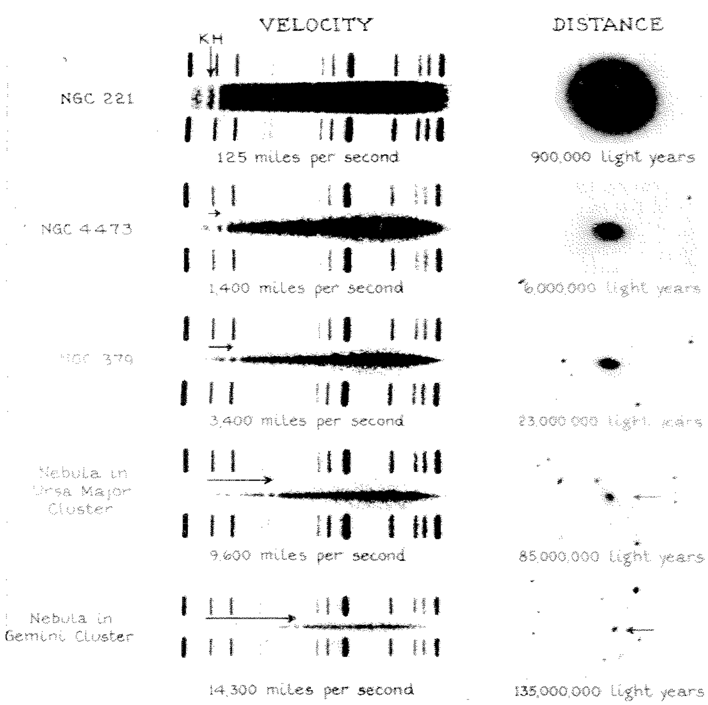
THE SHAPE OF SPACE

by James Gunn

There is only one universe, doing —as far as we can tell—rather simple things. But do we understand those things?



The velocity-distance relation for various extragalactic nebulae provides the basic observational evidence for the expansion of the universe. The greater the distance of a nebula from the earth, the greater the velocity of its recession and the more its spectra shift to the right, toward the red.

One of the most fascinating questions to which we can address ourselves is the ultimate one of beginnings and ends, and cosmology is largely devoted to seeking answers to that question. But, as it turns out, it is difficult to discover anything about the universe as a whole because of the extreme distances and faintness of the objects involved.

In assessing the findings of cosmologists, we must pay attention to a few powerful caveats. There is, first of all, the enormous conceit involved in our thinking that we can go to the laboratory and measure a few milliliters, liters, or even a few thousand kiloliters of something and then deduce laws to describe the evolution and properties of the universe as a whole. Second—and combined with our conceit—is a common trait, to which scientists are as prone as anyone else: our enormous powers of rationalization. We observe something in the universe, develop a theory that appears to explain it, and we're happy. But have we really explained anything? In the case of cosmological theories, it is very difficult to say. In other fields of science, it is not so difficult, because someone else can follow up and verify or disprove the results; the experiment can be repeated; other cases can be observed.

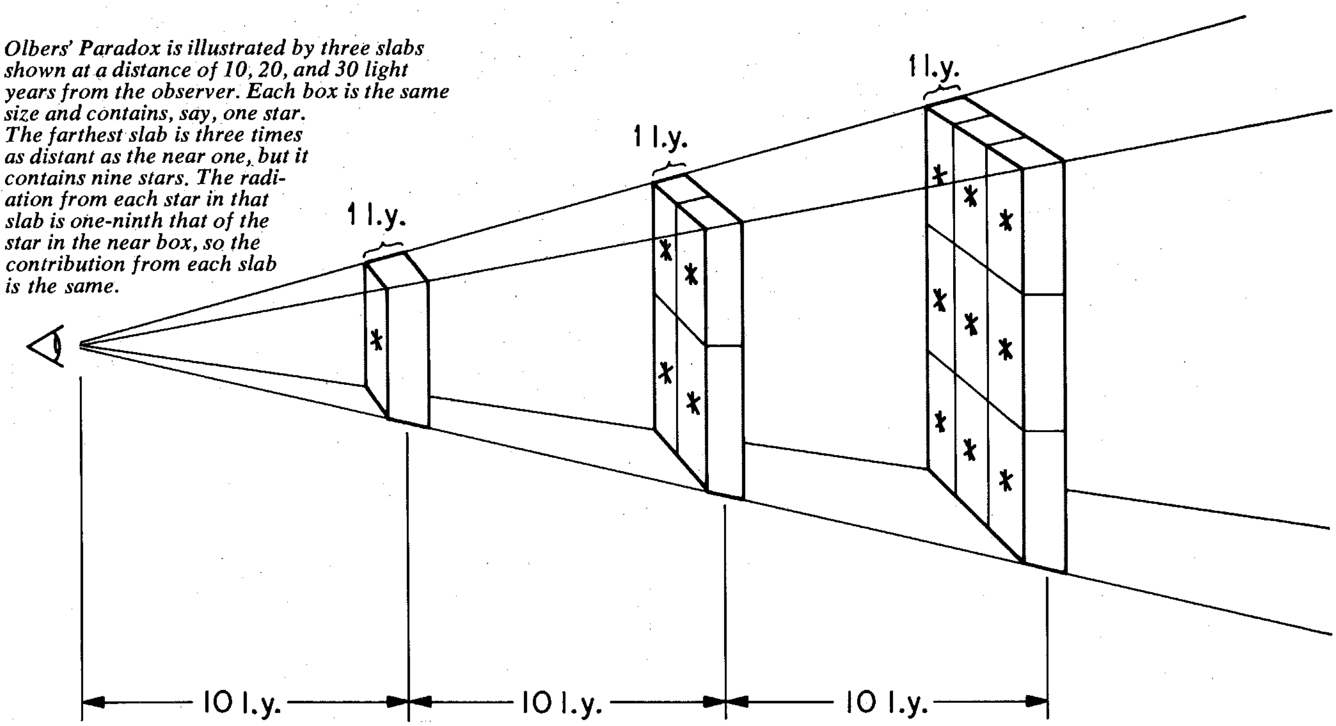
But there is only one universe. The universe is doing, as far as we can tell, rather simple things. But whether we really understand what is going on or not is quite impossible to say. We are not in a position to repeat the "experiment." We are not even in any position to perturb the experiment in any significant way to see how it behaves. That is perhaps not entirely unfortunate, but in any case it makes investigation rather difficult.

Despite these difficulties, the universe—or that portion of it we can see—presents a few facts for us to work with.

The sky at night is dark;
The universe is expanding;
The universe seems to be isotropic about us.

First, the darkness of the night sky is a very remarkable fact, for—as we shall see—it is not immediately obvious that it should be. Second, when we say the universe is expanding, we do not mean that the solar system, the stars in our galaxy, each aggregate of stars, and each galaxy are expanding. Rather, the galaxies—the so-called island universes that make up the "particles" of the universe—are receding from us and from each other. The distant ones are doing so at very great speeds. It is in this sense that the universe seems to be expanding, and it is manifested as the famous red shift in the spectra of distant objects. If we interpret the red shift as a normal Doppler shift—that there

Olbers' Paradox is illustrated by three slabs shown at a distance of 10, 20, and 30 light years from the observer. Each box is the same size and contains, say, one star. The farthest slab is three times as distant as the near one, but it contains nine stars. The radiation from each star in that slab is one-ninth that of the star in the near box, so the contribution from each slab is the same.



will be a decrease in the frequency, or an increase in the wavelength of light (toward the red) from an object receding from us—we can say that the reason for such a shift is that the most distant of these galaxies is receding from us at a significant fraction of the velocity of light.

The third fact with which the universe presents us—that it is isotropic around us—is in many ways the most remarkable fact of all. What we find is that the universe looks very much the same in any direction we choose to look. For example, if we pick two areas of the same size at random—one from the northern and one from the southern sky—we find about the same number of galaxies in one as in the other. The same sort of structures seem to exist in each.

Another phenomenon illustrates this isotropy even better: the background black body radiation of the universe. There turns out to be, in space, an infrared radiation field with a temperature of about 2.7 degrees Absolute, which comes to us from all directions. The temperature seems to be constant to within a tenth of a percent no matter where we look in space. It is generally believed that this radiation is residue from a very early epoch in the universe, the so-called Big Bang. So, not only is the universe highly isotropic now, it has been so for a very long time.

Let's look at the implications of these three facts in greater detail and see where they take us.

Around the turn of the 19th century, the German astronomer Wilhelm Olbers raised a very important theoretical question which highlighted a paradoxical contradiction between the evident "fact" of the darkness of the sky at night and the assumptions of cosmologists of the time which indicated that it should not be so. Now known as *Olbers' Paradox*, the question deals with the total radiation we would expect to receive on earth from all the stars in the universe. The radiation received from one star depends on both the energy it radiates in the form of light and its distance from us, because the flux of light (the rate of flow of energy across a surface) that we receive on earth is just proportional to the apparent area of the star. If the sun were moved twice as far away, its apparent size would be only half what it is now, and it would appear to be only one-fourth as bright. This means that four suns at twice the present distance would provide exactly the same amount of light as the sun does now. Thus, the light from the stars depends only on the area of sky covered by the stars and not on their individual distances. It can be seen from the drawing above that if stars fill an infinite universe uniformly—as it was believed in Olbers' time—the area of stars covering the sky in a shell, say, one light year thick is independent of the distance to the shell. But there are an infinite number of such shells, stretching to infinity, so the brightness should be infinite. But even

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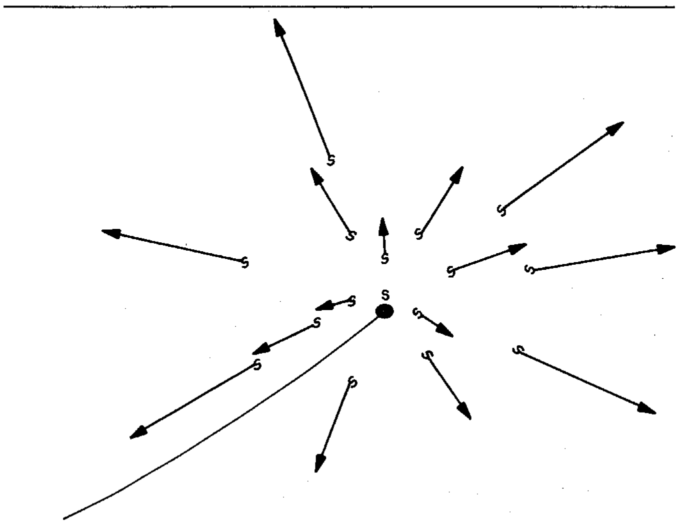
before "infinity" is reached, the sky would be covered with stars and the night sky would be as bright as the surface of the sun.

When Olbers introduced this paradox, it was the stars of our galaxy that were under discussion. But even though the stars in our galaxy do not go on forever in space, we are not released from the paradox. For when the stars of the other galaxies are taken into account, we are brought back to the same difficulty—whether the galaxies go on forever despite the vast distances separating them.

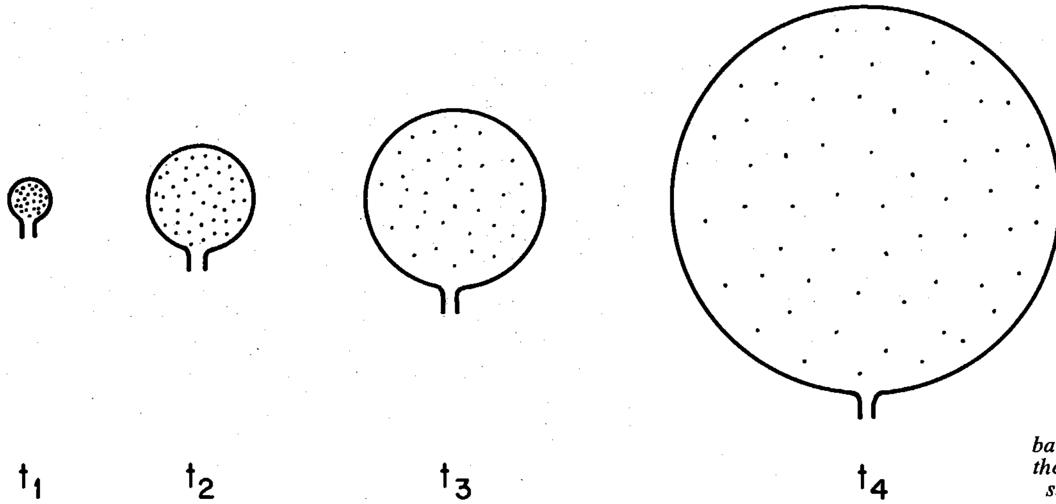
But, happily and obviously, the night sky *is* dark and not bright. It is the Big Bang, the expansion of the universe, that provides an escape from the paradox. Olbers assumed the universe was static, that the stars and galaxies were at rest. But if we assume an expanding universe, we find the light from the more distant of the galaxies is less than it would be if they were at rest at the same distance. The higher the recession velocity of a distant galaxy, the weaker the radiation received from it. Since the most distant galaxies move with successively higher velocities, the farther they are from the earth the less of their radiated light is received by us. We would receive almost no radiation from galaxies at the edge of the observable universe, which are moving at velocities close to that of light.



To an observer on earth, the galaxies in the universe seem to be expanding at a constant rate away from us. If we assume that there is no acceleration, we must also assume that all galaxies left the same neighborhood at the same time—now estimated to be about 19 billion years ago.

Also, as we look out into the universe, we are looking back in time as well. Since the speed of light is finite and we can look no further back than the Big Bang, the observable past of the universe is finite, not infinite. Albert Einstein's Theory of General Relativity predicts that we reach the epoch of the Big Bang at just the distance at which objects are receding at the speed of light, so the two solutions to Olbers' Paradox are really the same.

The expansion of the universe, while it explains many things, is the source of an exceedingly bothersome worry: Not only is the universe expanding, but it seems to be expanding away from *us* in particular. The relation between velocity and distance seems to be a linear one. If a distant object is receding at a given rate, an object twice as far away is traveling twice as fast. The velocity is computed by multiplying a constant (called the *Hubble Constant*) by the distance to the object. This formula, essentially, is *Hubble's Law*, and was first derived from measurements made by Edwin P. Hubble at the Mt. Wilson Observatory starting in 1923. Since everything is receding, it is reasonable to ask: If it has always traveled at the same speed, how long ago did a given galaxy leave the neighborhood of our own? It is easy to see that since the object twice as far away is traveling twice as fast, the time of departure for one and all is precisely the same. This leads to the conclusion that there was a time at which all these galaxies were on top of us, and some mighty event began their flight. We measure the age of the universe from this event. Our estimate of this number, the *Hubble Age*, has undergone tremendous revision in the last 20 years as new experimental data and observations have been taken into account. Currently, the best estimate for the age of the observable universe, determined in this way, is about 19 billion years. We are not aware of any structures that are older. Most are considerably younger. The earth, for instance, is estimated from geological evidence to be about 5 billion years old.



A two-dimensional universe that satisfies the Cosmological Principle and yet lets an observer see a universe as apparently expanding away from him is illustrated by the surface of an ordinary balloon. As time increases from t_1 to t_4, the balloon expands and the dots on the surface expand away from each other.

Now, the fact that we seem to be in the center of this show is a bit perturbing. The universe, as we have seen, seems to be highly isotropic around us; it also seems to be expanding about us. It all smacks philosophically of the anthropocentrism of the Middle Ages when the earth was assumed to be at the center of a set of crystal spheres beyond which there was heaven. It looks as if we are in some sort of highly privileged place from which we observe the universe. We have learned to regard this idea with some repugnance. Is there some way around this difficulty?

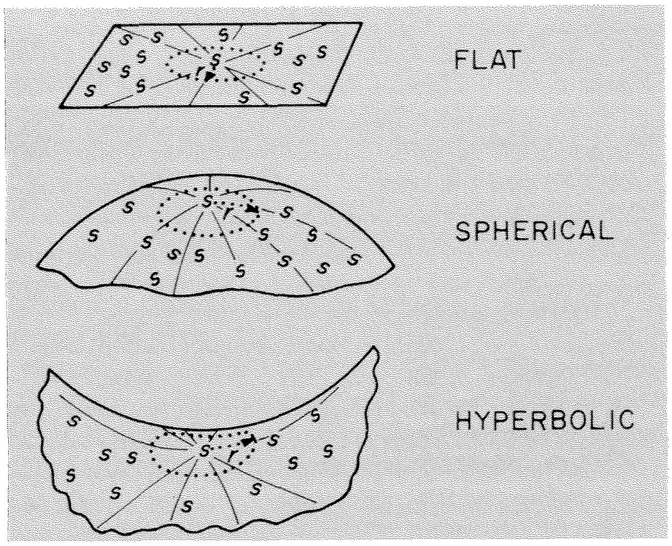
If the universe is at all reasonable and if we are not in a privileged place or a privileged time, the universe should obey a pontifical-sounding thing called by cosmologists the *Cosmological Principle*. It simply says: *We are not in a privileged place and no matter where you are in the universe you must see essentially the same thing.* A corollary to this principle is the idea of cosmic time. The Cosmological Principle doesn't make sense unless we can say *when* we should compare regions of the universe, since the universe expands and changes with time. What the principle must say is that there is a time by which everybody in the universe can set their clocks, so that if everyone looks at the universe at the same cosmic time they will see essentially the same things.

One thing is clear immediately, and that is that the universe—if it obeys the Cosmological Principle—cannot consist of a great expanding lump with us in the center, because an onlooker at the edge of such a lump would never see an isotropic universe. Can we, in fact, reconcile the observed isotropy—which seems to put us squarely in the center of something—with the Cosmological Principle?

That we can, at least in principle, remove ourselves from the center and still maintain these two ideas is illustrated by the diagram above. This shows a two-dimensional "universe" which seems to satisfy the Cosmological Principle—the surface of an ordinary rubber balloon. Suppose one glued little paper galaxies to its surface. To each galaxy its neighbors would appear to be receding as the balloon was filled. Each one thinks it is at the center of the expansion because every point on the surface of the balloon is just like every other point. As the balloon is inflated, the membrane expands uniformly, and the galaxies get further and further apart. This seems to satisfy both the isotropy and the observed expansion.

One could also do this with a very large two-dimensional flat rubber sheet being pulled with equal strength on all sides. If you had little galaxies glued to it, they would also expand uniformly away from each other. And unless you knew beforehand where the center of the sheet was, it would be impossible to locate the center by any kind of measurement you could make from a galaxy on the sheet.

So here are a couple of expanding two-dimensional cases which seem to satisfy the Cosmological Principle. What about the real world of three-dimensional space? What shape can it be? We know that ordinary space is describable by three dimensions, and the locations of points by distances in three directions—say, north, east, and up. Presumably, one can go as far as one likes in any one of those directions and keep going forever. Space seems to be Euclidian (flat) on any scale we can measure. But it is not at all clear that space is Euclidian on the very largest of scales. We haven't been able to penetrate space far enough to make any definitive measurements. We can, however, ask what the possibilities are mathematically. Infinitely many? Or is nature kinder than that? Do we have only a limited range of possibilities to select from? Any choice must meet the requirement that the space have precisely



The three possible shapes of space are flat, spherical, and hyperbolic (or saddle-shaped). They are illustrated here on two-dimensional surfaces, but the same possibilities exist in three dimensions—and in all higher dimensions.

the same properties everywhere and in all directions—it must be homogeneous and isotropic.

The restrictions are strong enough that one can show that there are only three possibilities for this kind of space. We have already dealt with two of these possibilities in two dimensions—flat and spherical. There is a third. In two dimensions the analogue is called a pseudosphere, which is a saddle-shaped surface. Flat space is infinite and unbounded. A line drawn in any direction extends endlessly; parallel lines remain parallel and the same distance apart. The curvature of this space is zero. Spherical space is finite but unbounded. A line drawn in any direction will not go on forever, but neither will it come to end. It will eventually close upon itself—making a circle. Parallel lines in spherical space, extended far enough, eventually meet. The curvature of this space is said to be positive. The curvature of the saddle-shaped space is negative. It is infinite and unbounded. Parallel lines drawn in this space eventually diverge from one another. Lines extend in any direction without end and without meeting themselves. Such a space is also called "hyperbolic."

These three possible shapes of space are illustrated, in part, in the diagram to the right. It turns out that these three kinds of space exist not only in two and three dimensions, but in all higher dimensions. If our space were nine-dimensional, there would still be only these three cases for the curvature.

If three-dimensional space were flat, it would obey all the laws of two-dimensional Euclidian flat geometry. One of these laws is that the sum of the angles of a triangle is 180 degrees. Another is that the circumference of a circle is 2π times the radius. In spherical space the sum of the angles of a triangle is more than 180 degrees, and the circumference of a circle is less than that of a circle in flat space. In hyperbolic space the sum of the angles is less than 180 degrees, and the circumference of a circle is greater than that in flat space. Space must be one of these three varieties if it is to obey the Cosmological Principle. Which one is it? And how do we determine it?

In two dimensions it is possible to determine whether an unknown surface is flat, spherical, or saddle-shaped by finding out whether the area of a circle drawn on it increases as the square of its radius, or whether it increases slower or faster. In three dimensions the question is how fast the volume of a sphere increases with its radius. The space is flat, spherical, or saddle-shaped according to whether the volume increases as the cube of the radius, or whether it increases more slowly or faster. The shape of the space we are living in, then, could in principle be determined by first counting the galaxies making up the universe at increasing distances out into space and then by seeing how this number changes with distance, since the Cosmological Principle demands that the number of galaxies per unit volume be uniform.

But we cannot determine the volume (and therefore the shape) of space directly because we cannot determine distances with sufficient accuracy. We must seek other means. We must again consider the expansion of the universe, investigate the forces acting on the expansion, and determine how they relate to the density of matter and the "total energy" in the universe.

Let's take a chunk of the universe, make a bubble, and talk about the behavior of the stuff within this bubble. If we make the bubble small enough, we certainly can explain its behavior by the everyday physical laws we know. It is a curious consequence of the Cosmological Principle that this same bubble must be typical of the universe as a whole, since every piece of the universe is like every other. Thus if one understands this bubble, one understand the whole universe—if one knows how the bubbles fit together.

Consider such a bubble. What makes it expand? One can show quite convincingly that the forces that might drive the expansion are probably much too small to do the job. The universe seems to be expanding because it once got going that way and has been coasting ever since. There seem to be no forces that could even significantly alter the expansion, except for a very important one—gravity. How is this force affecting the universe?

Again, there are three choices; three things that can happen in a universe acted upon by gravity alone; three states of energy.

The energy can be zero. A good analogy is the case of a rocket being launched from the surface of the earth. The rocket can be launched at precisely the escape velocity of the earth, so that it climbs up very slowly and eventually gets as far away as you like. But all the while it is slowing down more and more, so that it reaches infinity with zero velocity.

The energy can be positive. You can push the rocket out a bit faster than the escape velocity. That means that once it gets beyond some point it really no longer feels the gravitational field of the earth very much. And so it continues from there to infinity with essentially constant velocity.

Or the energy can be negative. You could fire the rocket with not quite enough energy to escape the earth's gravitational pull. In this case it will reach some maximum distance, pause, return to the earth, and crash with the same speed as that with which it was launched.

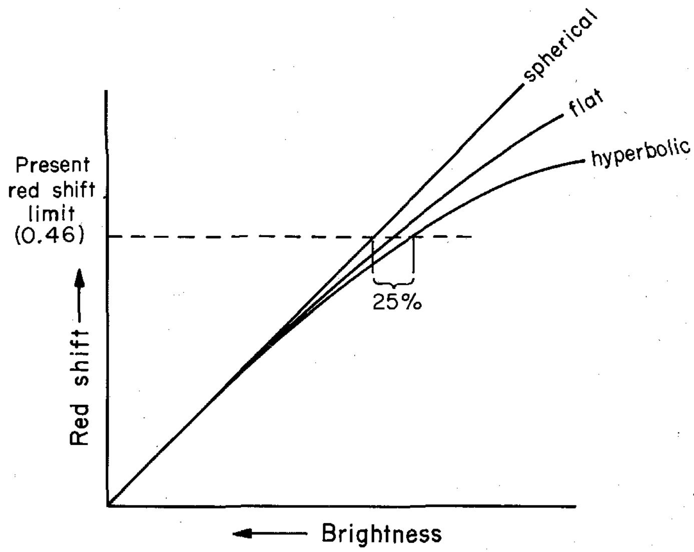
In a similar manner, if the total energy of the universe is zero, the expansion will continue, but it will become slower and slower as time goes on. If the energy is positive, the universe will also keep expanding, but eventually the effects of gravity will become negligible and it will fly apart at a constant rate. If the total energy is negative, gravitation will eventually stop the expansion, and the universe will collapse again in a backward rerun of the Big Bang from whence it came. Gravitation always slows the expansion—the question is simply whether it "wins" or not in the end.

Einstein's General Theory of Relativity predicts that these energy considerations are intimately connected with the shape of space (how the bubbles fit together). It turns out that if the universe has negative energy, space is spherical. Such space has finite volume. There is only a finite amount of stuff in the universe. You could go around the universe and eventually come back to where you were. (You would have to go faster than light to do it, however.) In this case you may recall, the universe will ultimately collapse, so the whole thing is finite in both space *and* time. If the energy is zero, space is flat. If the energy in the universe is positive so that the expansion eventually proceeds unhindered, it turns out space is hyperbolic (saddle-shaped).

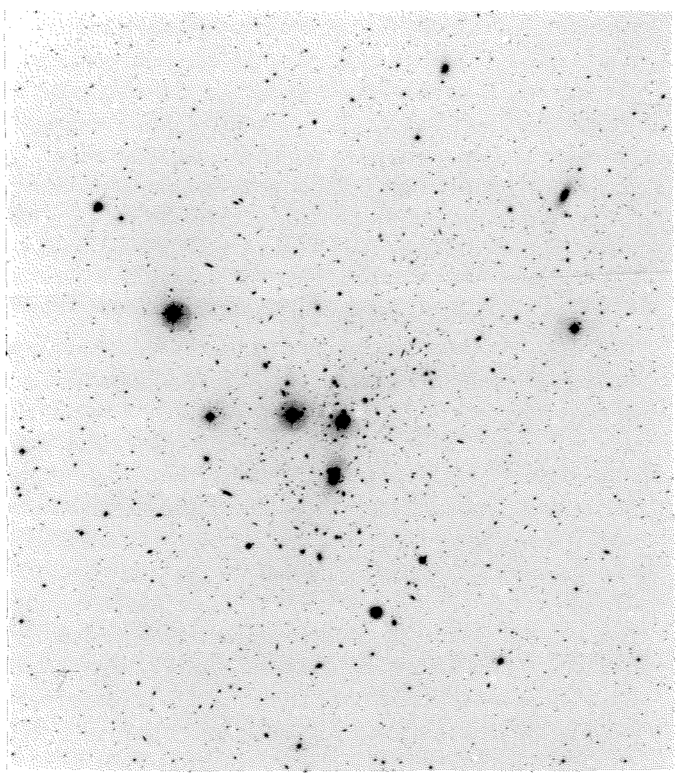
If you wish to believe these predictions of General Relativity, there are several ways to get at the shape of space.

One possibility is to look at the density of matter in the universe. This is a very difficult thing to measure, but would give the answer if we could do it. The higher the density, the stronger the gravitational force which decelerates the expansion. Knowing the density, the law of gravity, and the expansion rate, we can calculate the energy and hence get the curvature. But how does one get the density? One direct way is to use approximately known masses of the galaxies we see. We can then conceptually smear this mass out and find the mean density. That density is equivalent to about 1 atom for every 10 cubic meters. This value for the density implies that the total energy of the universe is strongly positive. Hence, the expansion would continue forever. The universe is infinite and unbounded, and hyperbolic in shape. We must, however, regard this result with a great deal of caution. We are measuring only the density of the matter we can see, and we must assume that there may be more that we do not see for some reason.

There are some people to whom it is philosophically very important that the universe be closed, that the energy be negative, that the universe began at some time with a violent bang and will end the same way. This concept makes a neat little bubble in space-time. There have been some quasi-scientific reasons for the concept, but I don't think they are at all compelling. To have a closed universe of this sort, one needs a density of about 3 atoms for each cubic meter—about 30 times as much as the galaxies contain. This is only possible if, for example, there is a diffuse gas of about this density spread out between the galaxies. Various people have looked very hard to find this material but with little success. On the other hand, it has been very hard to prove it is not there, though some progress is being



The diagram above shows how the received radiation from a "standard bulb" varies with red shift for three examples of space curvature. At the present red shift limit for super-giant galaxies (.46 of the velocity of light), the differences are only 25 percent—too small to be reliably determined in the face of statistical and other uncertainties.



Possible "standard bulbs" against which astronomers can compare distant galaxies are super-giant galaxies such as those within the Coma cluster of nebulae. Quasars will not do because we know so little about them. Super-giants, by contrast, are reasonably similar and of known brightness.

made in this direction. It now appears that there probably is not enough matter to reverse the expansion—but this result is very tentative as yet.

Another indirect way to get at the answer—the one which will probably eventually yield the best data—is to look at the relation between recession velocity and distance. We have seen that this relation is linear, but it turns out this is true only for nearby objects. And for a very simple reason. As we look out in distance, we also look back in time. Gravity has been slowing the universe down; so as we look out, we look back to eras when the expansion was faster than it is at present. Thus, we can measure the rate of slowing down, and, hence, determine the gravitation and the energy. This technique cannot be used directly, for we have no sufficiently good way to measure the distance. We can measure the brightness of distant objects, and, of course, their red shifts, to obtain their velocities. If we know how luminous a source is—its total light output, the "wattage of its light bulb"—we can deduce its distance. If we have a set of "standard bulbs," all of precisely the same power but at different distances, we can expect to see a relation as is shown in the figure above left. For sufficiently distant objects the difference in apparent brightness for different kinds of space is quite appreciable.

What does nature furnish us with that we can use for our standard bulbs? It was hoped that the quasars would do, for they are very bright and can be seen from enormous distances. But they seem to come in all "wattages," and furthermore, there seems to be no way to tell an intrinsically faint one from a bright one. The next best things are super-giant galaxies, which for reasons we do not understand at all seem to be remarkably alike. Furthermore, they are easy to find, since they are always the brightest member of a cluster of galaxies, such as the Coma cluster in the photograph to the left.

These brightest of galaxies are, unfortunately, faint compared to quasars, and so it is very difficult to observe them at great distances. The most distant yet studied is receding at about one-third the speed of light and is not far enough away to tell reliably which curve is the correct one for the shape of space.

Development of light detectors is a field in which technology is advancing rapidly, however, and within the next few years we should be able to study objects which are twice as distant as any we have yet seen.

We may yet know the shape of space.