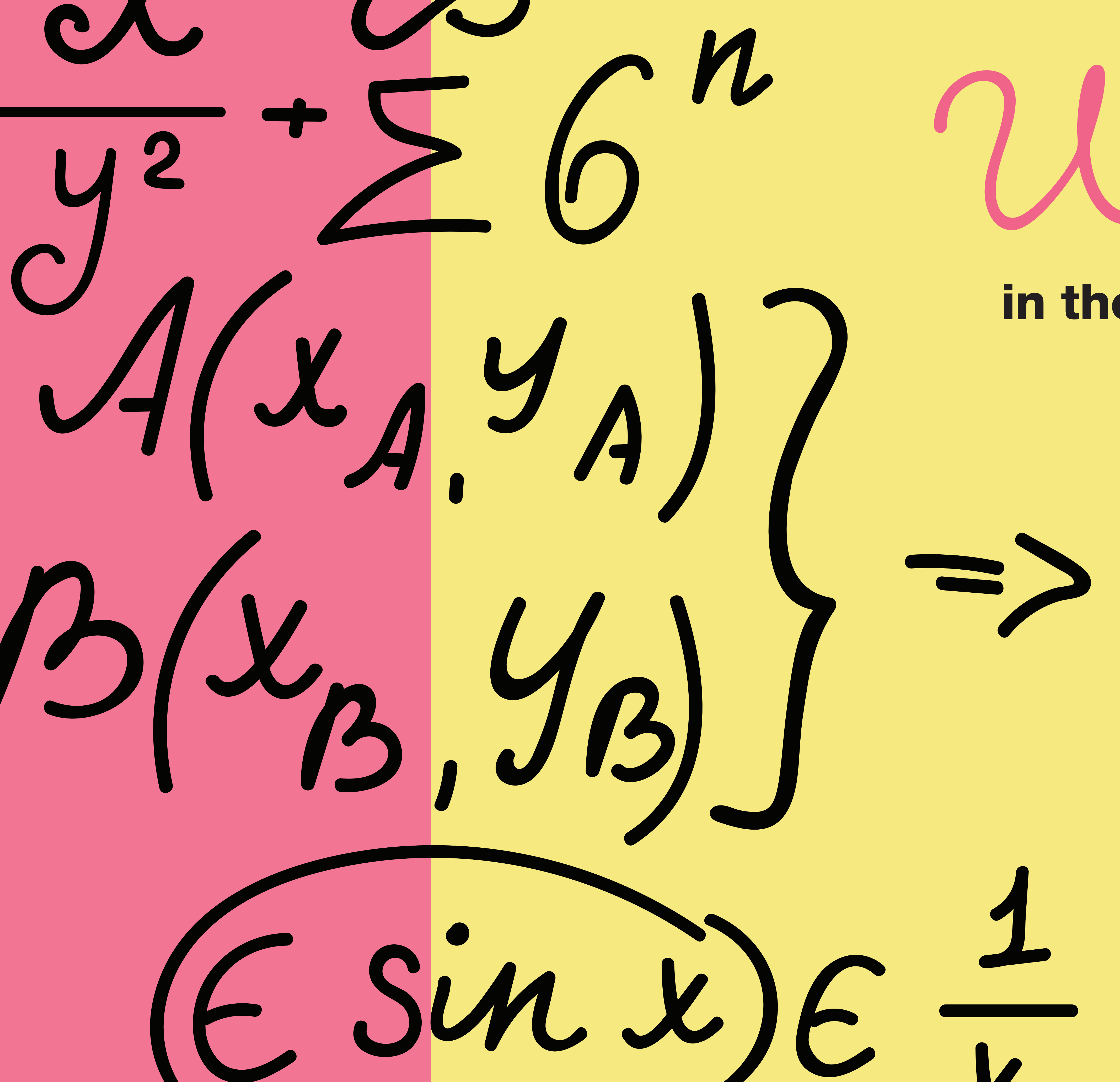


Writing in the Language of Math

By Whitney Clavin

From chalk to software code, mathematicians and scientists use a variety of methods to express equations and formulas, and they have different ideas about the meaning behind their numerical prose.

In decades past, when researchers had to bang away at typewriters to write up their scientific papers, they would often hit a snag. The machines, including the IBM Selectric series popular from the 1960s to 1980s, did not contain keys for mathematical symbols, such as the long "S" used to represent integrals in calculus. When it was time to type in an equation, the researchers had to hunt for golf ball-sized silver spheres that contained the proper characters to snap into the typewriter. Some sought workarounds to avoid the hassle.

"I didn't have the patience to use the IBM Selectric and switch out the balls," says John Preskill, the Richard P. Feynman Professor of Theoretical Physics. "When I was an undergrad, I typed my senior thesis but left space for the equations and wrote them in by hand. For my PhD thesis, a technical typist typed out what I had written in longhand."

Today, typewriters have given way to personal computers and other modern technologies, making it easier for mathematicians and scientists to communicate their mathematical equations to collaborators and the rest of the world. Researchers share their formulas on virtual whiteboards, snap pictures of equations with their smartphones, and write out equations in documents using a much-heralded software program called LaTeX (pronounced lay-tech).

Across the Caltech campus, scientists and mathematicians have various methods for writing in the language of math. Some have gone digital, while others still prefer the gritty, tactile nature of chalkboards. Fernando Brandão, the Bren Professor of Theoretical Physics, says he has switched over to digital pens and pads to write out equations. "I used to have equations written all over papers that were everywhere," he says. "Now I save paper and trees."

Not all researchers see these handwritten symbols and software codes the same way. Some call math the language of nature, while others describe it as a tool for abstract reasoning. Nevertheless, the researchers agree that mathematical notations allow them to reveal hidden patterns and structures in our world related to stock markets, computers, black holes, and even living beings.

"Math lets you encapsulate a lot of ideas at once. It lets you find incisive solutions that get to the heart of a problem," says Tom Hutchcroft, professor of mathematics. "Imagine playing a game of chess and having to write a paragraph about each of your moves. This is what it would be like to do math without the equations."

Ancient Arithmetic

The first written mathematical notations date back thousands of years and can be found carved into clay tablets, stones, and wood. The ancient Egyptians depicted numbers ordered in powers of 10 with symbols, or hieroglyphs, such as lotus flowers, fingers, and frogs.

The modern mathematical symbols we know did not come into fashion until the Renaissance, which began in the 14th century. Before the symbols were introduced, the ancient Greek mathematician Archimedes, for example, derived the value of pi by using hexagons to approximate circles but did so without the use of the pi symbol, π, which was not introduced to represent the mathematical constant until the early 18th century.

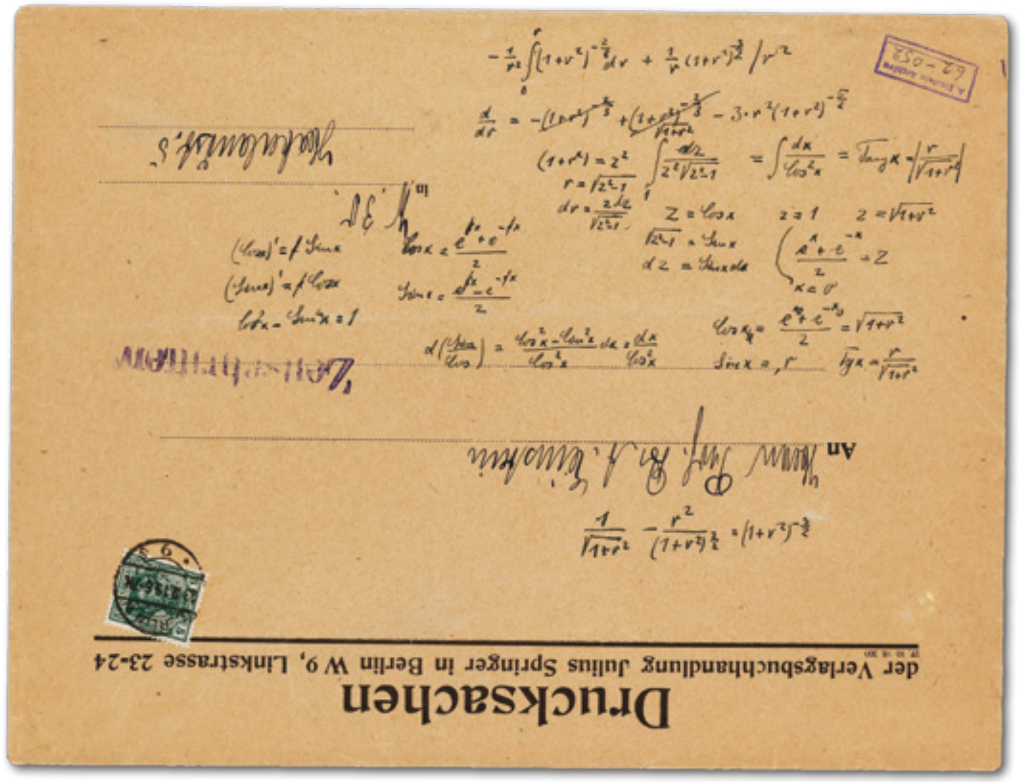
As math progressed, so did the need for mathematicians and scientists to write down their equations. The advent of the typewriter in the 1870s made the job easier, but it was still a cumbersome task. In fact, many published papers, including those by Albert Einstein, who was a visiting professor at Caltech on three occasions, and longtime Caltech physics professor Richard Feynman, contain handwritten equations sandwiched between typed text.

Einstein actually wrote on the front and back of envelopes to carry out his calculations, explains Diana Kormos-Buchwald, Caltech's Robert M. Abbey Professor of History and director of the Einstein Papers Project. That archive includes one such envelope from 1919 on which Einstein jotted down unidentified calculations. "This was a year after World War I ended, and there was a terrible paper shortage. People reused all paper before recycling was invented," she says.

LaTeX Revolution

Preskill and other Caltech researchers recall the invention that finally relieved researchers of the burden of switching out typewriter balls.

Right: An envelope featuring handwritten calculations made by Albert Einstein in 1919. **Below:** A Babylonian clay tablet from c. 2000 BC with carved arithmetic tables.



Theoretical physicist Monica Jinwoo Kang enjoys writing equations on a blackboard with chalk made by the Korean brand Hagoromo.

Beginning in the late 1970s, Donald Knuth (PhD '63) developed a typesetting language called TeX, which allowed researchers to more easily type out mathematical symbols. Leslie Lamport, a computer scientist then working at SRI International in Menlo Park, California, took TeX a step further to create the document-preparation system known as LaTeX, which uses the language of TeX. For example, to type the symbol π into a document, a researcher would write the command "π" into the LaTeX program.

"LaTeX has changed the way we communicate with each other," says Professor of Theoretical Physics Xie Chen, who, like many other scientists, uses the TeX language to denote mathematical symbols when writing technical papers and even emails to colleagues. Sometimes, she says, they do not even bother to translate the codes back into mathematical symbols because the codes are so ingrained in their heads.

Smartphones have also changed the ways scientists communicate math; it is easy to snap pictures and send photos of formulas to one another. Virtual whiteboards have become more popular too, says Sergei Gukov, the John D. MacArthur Professor of Theoretical Physics and Mathematics. Gukov and many other researchers took part in online math communities during the pandemic that allowed them to connect with one another virtually in classroom-like settings and tackle various math and science problems. These communities were developed by the National Science Foundation–funded American Institute of Mathematics, or AIM, which will move its headquarters to Caltech in 2023. "The virtual whiteboards were key to these online communities and felt very much like real blackboards," Gukov says.

The rise in popularity of videoconferencing has added yet another tool for mathematicians to communicate with one another, Preskill says: "Sometimes on Zoom, you just write an equation on paper and hold it up to the camera."

Analog Beauty

Even with the advent of new communication tools, some researchers prefer to go old school. Monica Jinwoo Kang, Sherman Fairchild Postdoctoral Scholar Research Associate in Theoretical Physics, says she loves to write equations on her blackboard. "The fact that the blackboard is so big helps me think more clearly," she says. She even has a favorite chalk: a Korean brand called Hagoromo.

"The difference between other chalks and Hagoromo is striking: the grip, the feel, and the effectiveness," she says. “The chalk is a bit thicker and holds amazingly while preventing any scraping sounds on blackboards. Also, the chalks write effortlessly without applying much pressure or force and without making a mess.”

Matilde Marcolli, the Robert F. Christy Professor of Mathematics and Computing and Mathematical Sciences, likes to create colorful watercolor pictures by painting over her equations. The creative process helps her focus on her mathematical research, she says. “It’s a kind of meditation, in a sense, where you are looking at the calculations you just wrote and thinking them over; the process of painting helps in formulating connections in the mind. It’s like a form of synesthesia, where mathematical thoughts come with colors and shapes attached to them.”

Rich Abbott, an engineer for the Laser-Interferometer Gravitational-wave Observatory (LIGO), has a similar affinity for handwritten equations, and he always writes them by hand on official engineering documents. “It’s beautiful to see the equations written out,” he says. “They are more human and compelling that way. You want to understand them.”

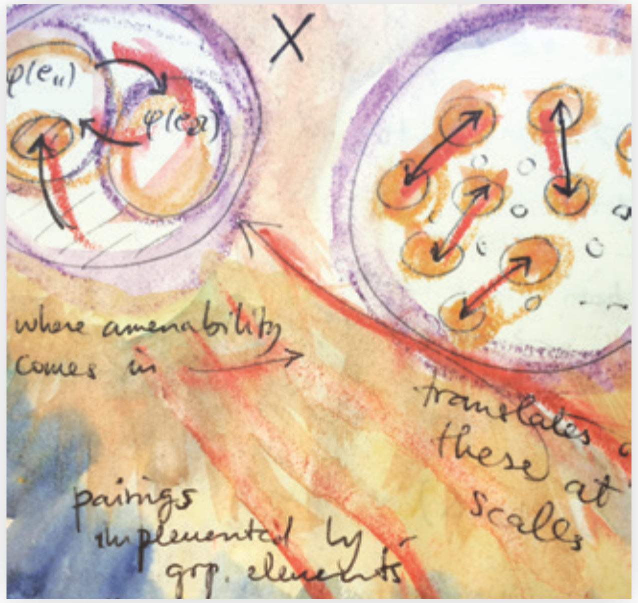
A New Lexicon

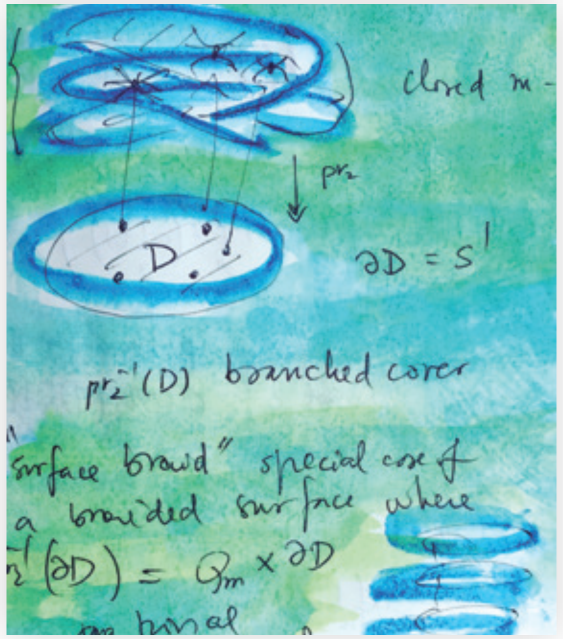
Math can be communicated in words too. When Archimedes explained to his peers how he calculated the value of pi, he did not have a language of mathematical notations to pull from. Instead, he used pictures and words. “It’s hard for me to fathom how Archimedes did that,” Hutchcroft says. “It’s one thing to have a personal understanding of how you think things work but quite another to explain your ideas to people.”

Today, words are still a key part of mathematical research. Omer Tamuz, professor of economics and mathematics, is intrigued by the words people invent to explain math.

“Math is more than equations,” he says. “Instead of writing ‘x=0,’ you can write ‘x vanishes.’ Thousands of words are invented to represent objects or functions in math. Sometimes, the invented words are clever and help you remember the meaning.”

Consider the word “matroid,” Tamuz says. A matroid is similar to a mathematical object called a matrix and ties together different concepts such as vectors and graphs. “Somebody made up this silly word, but it’s useful because you remember it, and it makes sense,” he says. “Pointless topology” is another math term Tamuz likes because it’s a “good play on words.” Pointless topology refers to a way of thinking about the properties of a geometric object without points, he says, adding that he considers it humorous to label an invented math concept as “pointless.”



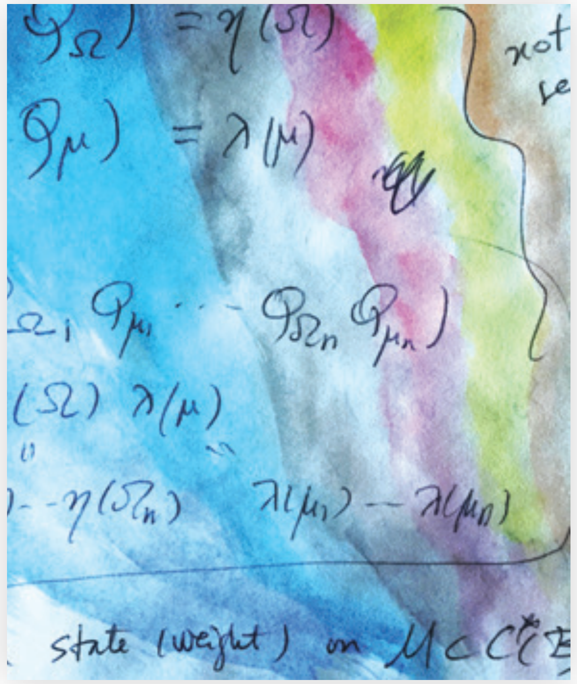


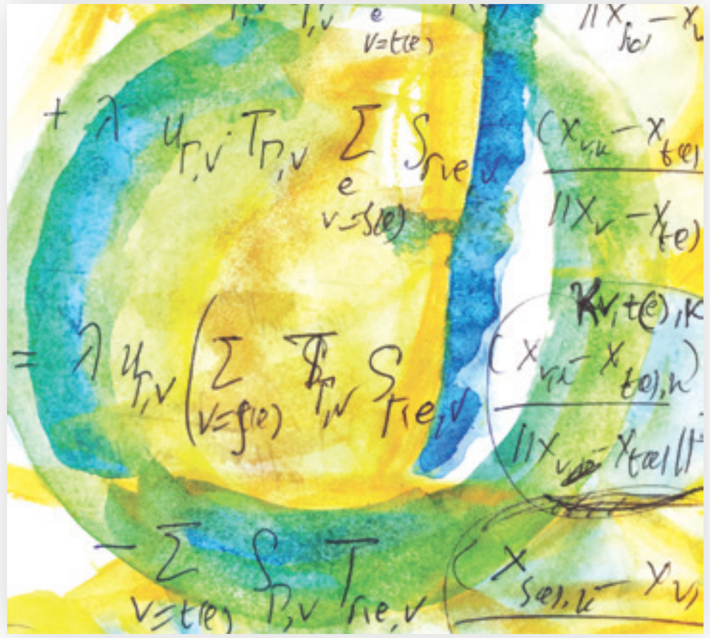
Right: Watercolor paintings by mathematician Matilde Marcolli that feature her written equations.

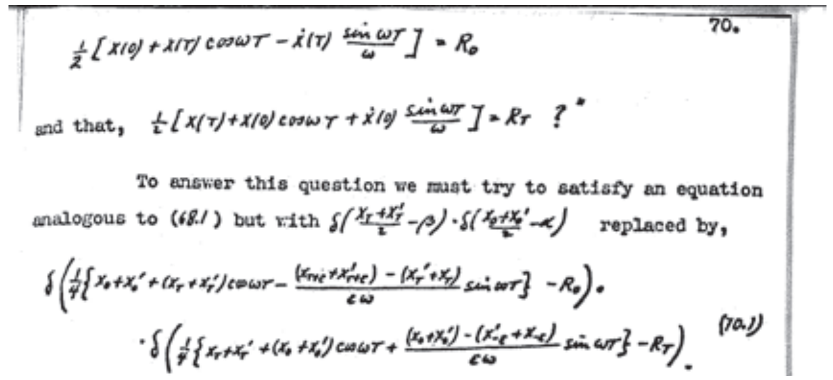
What Does Math Mean?

If math can be expressed in words, what does this say about the true nature of math? Are mathematical symbols merely human inventions used for expressing abstract ideas, or are they part of a fundamental language of the universe that exists independently of us?

Christopher Hitchcock, the J. O. and Juliette Koepfli Professor of Philosophy, says there are two main camps when it comes to the meaning of math and numbers. The first group follows ideas put forth by the ancient Greek philosopher Plato, who believed mathematical objects are real and possess identities that exist beyond ourselves.







Above: Richard Feynman handwrote equations in his 1942 PhD thesis at Princeton.

“Plato thought that mathematical truths would be true even if we didn’t exist,” explains Hitchcock. “Think of a line in geometry that is infinitely straight and perfectly thin. It’s not a physical object: there is nothing in our world that is like this. But it still has an existence independent of us. Another way to think of this is to ask: If there are an odd number of trees in the forest and nobody is there to count them, is the number still odd? Plato would say yes.”

Many scientists agree with this Platonic view. Physicists in particular often describe math as the language of nature. Feynman, the late Nobel laureate who taught physics at Caltech for nearly four decades until his death in 1988, said in a lecture at Cornell University in 1964: “To those who do not know mathematics, it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature. … If you want to learn about nature, to appreciate nature, it is necessary to understand the language she speaks in.”

But as Hitchcock explains, other philosophers believe math is merely an invented tool of science and not an inherent part of our reality. Hartry Field of New York University has gone so far as to reformulate Newtonian mechanics without references to numbers. Field argues the structures described by math are fictitious and not literally true. To understand this viewpoint, Hitchcock gives the example of three plates on a table. It is possible, he says, to describe the plates without using the number three. You might describe each plate and point out that they are different from one another. In a similar way, you could say that each plate has six cookies. You could use multiplication to figure out there are a total of 18 cookies, but you could also derive a sentence describing each cookie and say they are different from one another without using the number 18. Exercises like this, he says, are meant to illustrate that math is simply a tool.

“You can use logical reasoning to explain things without numbers,” Hitchcock says. “But as the numbers get higher this becomes increasingly difficult. The numbers make the calculations much simpler.” Hitchcock himself believes in a more Platonic view of numbers. They are real, he says, but “in their own peculiar way.”

Whether or not numbers and math are real may remain an open question, but one thing is clear: mathematical writings scribbled on papers, digital pads, and coded in LaTeX continue to elucidate the wonders of our world.

Rob Phillips, the Fred and Nancy Morris Professor of Biophysics, Biology, and Physics, recalled in an article from the Summer 2021 issue of *Caltech* magazine that one of his grad school teachers had a blackboard on his door and used to tell him that if he had something to say, he should write it down in equations. “The language we speak in my lab is not English but math,” Phillips said. [C]