Origami: Complexity Increasing

by Robert J. Lang

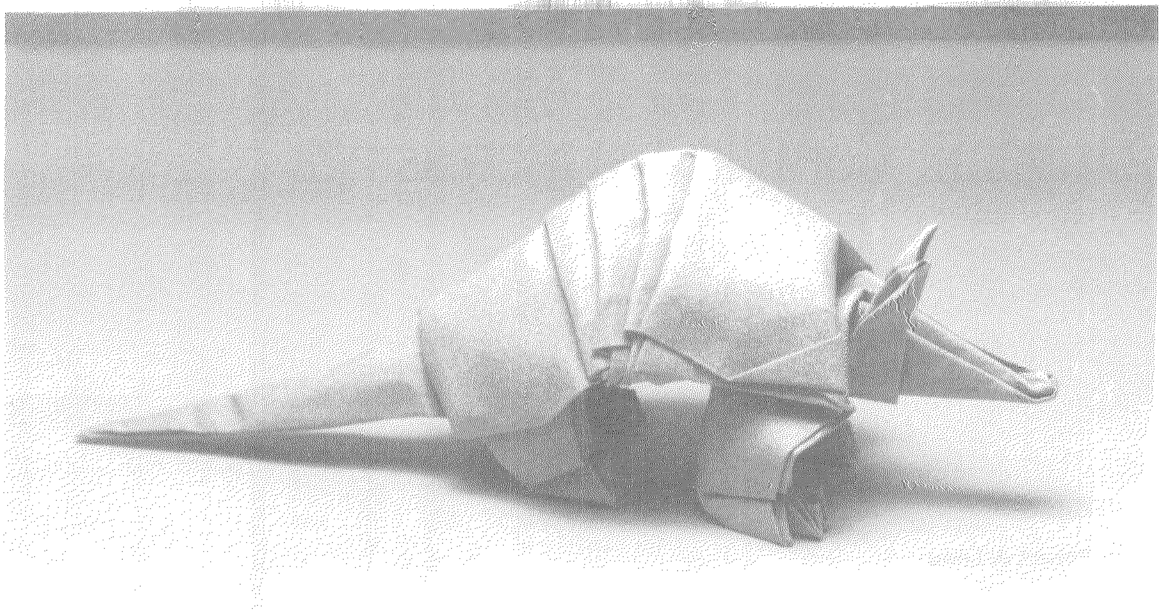
Lang's "Black Forest Cuckoo Clock" is made from a single sheet of paper—a 10:1 rectangle. The model contains about 200 meters of creases and takes 4 to 6 hours to fold.

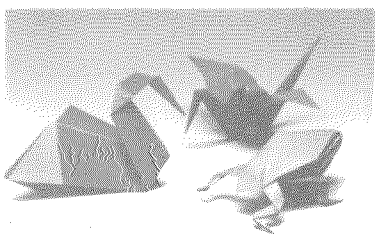
Sitting on the sill of a window at the Jet Propulsion Laboratory are three small figures. They look somewhat incongruous among the piles of journals, computer diskettes, books, and glassware scattered about; they are a man playing a violin, a man playing a string bass, and a man playing a grand piano. Each has been folded from a single sheet of paper. They are examples of an art called origami, which is a Japanese word meaning "folded paper." The art originated in Japan, where it has been an integral part of religious ceremonies for some 1,500 years. There, folded paper streamers, called *gohei*, and paper human figures, called *katashiro*, are placed in Shinto shrines to receive the temple deity. The three instrumentalists did not originate in Japan, however.

They are a result of the peculiar fascination origami has for at least one scientist, in this case, myself. The appeal is due to many things. Origami is a game; it includes topology, it requires visualization. There are pleasant symmetries involved in the transformation of a square of paper into a bird or flower, a transformation that may be very elegant. Within origami, we see the apparent creation of something from nothing, order from disorder, entropy reversed. These aspects of paperfolding have attracted thousands of people to the art in recent years. They have inspired explorations into many of the mathematical and physical properties of folded paper. They have drawn me into origami and held my interest for some 20 years, and they are ultimately responsible for the three instrumentalists I designed that now sit above my desk.

The trail that ends at a physicist's window sill began in China around the year 100 A.D. with the invention of paper. By the 4th century paper had traveled to Japan; in the centuries that followed, the secrets of its manufacture spread around the world. Somewhere along the way, someone discovered that paper could be folded into a variety of interesting shapes. This pastime was established in Japan by 600 A.D., and was called origami, from the Japanese words *ori*, meaning "folded," and *kami*, meaning "paper." It was, and still is, a folk art. Mothers teach simple folded designs to daughters as they have done over the centuries. The traditional origami designs encompass some 100 simple toys, abstract shapes, and representations of birds, animals, and flowers.

That description of the art would have been accurate 50 years ago. At that time, all of the different origami designs in the world could have been cataloged on a single typed sheet of paper, had anyone the inclination to do so. No model would have run over about 10 or 20 steps. Most could be folded in a few minutes, even by a novice. This is no longer the case. Today, in books, in journals, and in personal archives, the number of recorded origami designs runs well into the thousands, and many of the most sophisticated designs have more than a hundred steps and take over an hour for an experienced folder to produce. The growth in the number of models owes its existence to a burgeoning worldwide interest in paperfolding, but the growth in the complexity of designs is due to something else: 1,500 years after its invention,





Traditional Japanese origami designs included simple representations of birds and animals. More complex animal designs, such as Lang's "Armadillo" (top) evolved only in the last 25 years.

origami was discovered by science.

Science is attracted to a challenge, and the challenge of origami lies in its rules. Simply put, origami is the art of folding uncut squares of paper into decorative objects. That is the modern definition. Many traditional Japanese designs, both ceremonial and recreational, were quite liberal with cuts, however. Origami designs from the *Kayaragusa*, a collection of 49 examples of ceremonial and recreational origami dating from 1797, included many examples with cuts, and many other traditional Japanese designs use multiple sheets of paper. Modern folders are more conservative with cuts, more liberal with shapes. Some use rectangles, some triangles; some allow cuts that don't actually remove paper; others allow more than one sheet but no cutting at all. There is no agreement among the world practitioners on absolute rules about multiple pieces, or unusual shapes, or cutting, but there is widespread agreement that there is a purest form of origami, which is: one square, no cuts.

Cuts or not, for 1,500 years, the origami repertoire remained essentially static. Part of this lay in the way it was passed on. Word of mouth does not allow complex designs to survive more than a generation or two. Then, too, original designs were not encouraged. When, in the 1920s, a metalworker named Akira Yoshizawa began to invent new designs, he was not supported in his work. Fortunately, he persevered. Yoshizawa, now considered the father of modern origami, publicized his own work through exhibitions and books. Through his efforts, the art caught on around the world, and an era of new designs began that continues today. The designs and designers multiplied. English-language books began to be published, further accelerating the spread of knowledge. Then in the 1970s, a new breed of paperfolder began to appear. Mathematicians, scientists, and engineers were attracted to origami and began to approach it in a new way: not as a form of artistic expression, but as a source of technical challenge.

Before this time, the typical way for a person to invent a new design was to fold one of a handful of basic folds, or "bases," and play with it until it began to bear a resemblance to something. Throughout the first three-quarters of the 20th century, origami grew by trial and error. Unsuccessful attempts were discarded, successful ones recorded. The number of designs grew, but the sophistication of designs remained relatively steady.

The design of an origami model may be broken down into two parts; folding the "base," and folding "details." A base is a regular geometric shape that has a structure similar to that of the subject, although it may appear to bear very little resemblance to the subject. The detail folds, on the other hand, are those folds that transform the appearance of the base into the final model. The design of a base must take into account the entire sheet of paper. All the parts of a base are linked together and cannot be altered without affecting the rest of the paper. Detail folds, on the other hand, usually affect only a small part of the paper. These are the folds that turn a flap into a leg, a wing, or a head. Converting a base

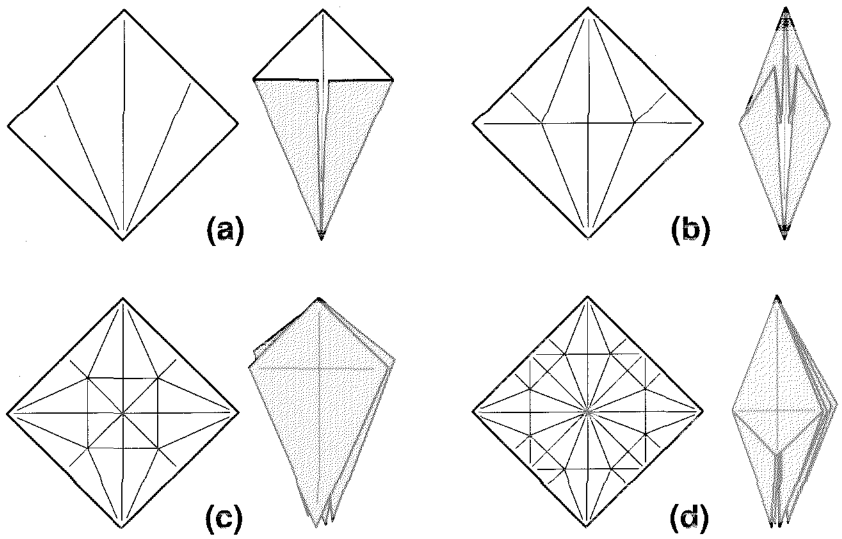
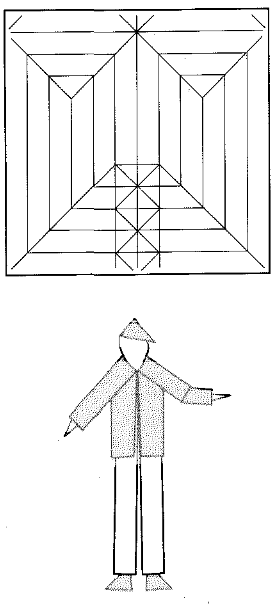


Figure 1: (right) The "classic" origami bases and their *crease patterns:* (a) Kite Base; (b) Fish Base; (c) Bird Base; (d) Frog Base. (Not to the same scale.)

Figure 2: (below) Crease pattern and model of Neal Elias's "Man in Black and White." All creases in the base are either vertical, horizontal, or at 45°.



into an animal using detail folds requires tactical thinking. Developing the base to begin with requires strategy.

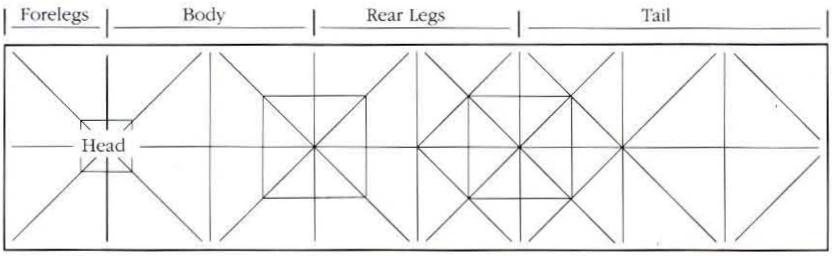
By the mid-1960s, four bases were in widespread use (Figure 1); in English-speaking countries, they are called the Kite Base, the Fish Base, the Bird Base, and the Frog Base. (In addition, there are two other shapes commonly called bases—the Preliminary Base and the Waterbomb Base—that are precursors to the Bird and Frog bases.) All four, dubbed the "classic" bases, were known to the Japanese for over a hundred years before origami made it to the West.

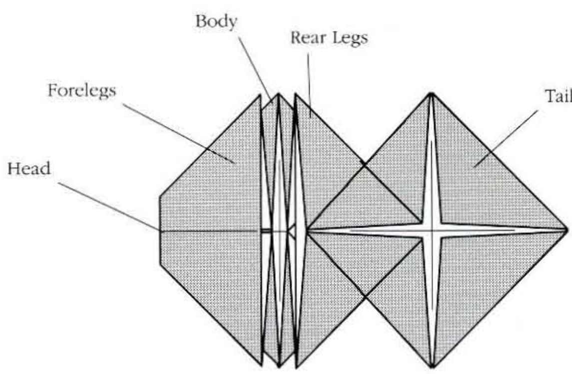
Major points on a base get turned into major appendages of a final model. The Kite, Fish, Bird, and Frog bases have, respectively, one, two, four, and five large points and one, two, one, and four smaller points. To fold an animal, you need to start with a base that has the same number of points as the animal has appendages. A simple fish has two large points (head and tail) and two small ones (pectoral fins), which is why the Fish Base is so appropriate and so named. The average land-dwelling vertebrate has five major points (four legs and a head), which pretty much stipulates the Frog Base and rules out a tail. The point on the Frog Base that is in a position to form a head is thick and difficult to work with, however. One of the four points of a Bird Base would be easier. But to use a Bird Base to fold a four-legged animal, you would have to represent two of the legs (usually the rear legs) with a single point. In the 1950s and 1960s, there were a lot of three-legged origami animals running around.

Not only were the four classic bases widely known by the mid-1960s, but their shortcomings were known as well. Brilliant advances in detail folds had been developed, techniques to give the appearance of separate legs, but no actual legs were forthcoming. Clever use of different colors (from opposite sides of the paper) gave the appearance of multiple subjects from the same sheet of paper, provided those subjects had no long appendages. There were isolated successes—an elephant with legs, ears, and tusks, made by folding the corners of a square to the center before folding a Bird Base—but in general, no systematic method existed for making complex subjects.

In 1963, an amateur magician and an engineer broke from the traditional bases. Neal Elias and Fred Rohm began their explorations on a challenge: to make a working Jack-in-the-box. They hit upon the technique of limiting all creases in the base to a series of parallel creases that divide the paper in one direction; a set perpendicular to those; and a set at 45° to the others (Figure 2). With the paper so creased, it may be collapsed on these creases into a variety of regular shapes with varying numbers of points and flaps. By this means, far more complicated structures were possible than with conventional folding techniques. Because the paper is initially pleated and intermediate steps in the formation of a base are assemblies of boxes, the techniques are collectively referred to as box-pleating.

Box-pleating may be used for models made from squares, but the techniques are especially





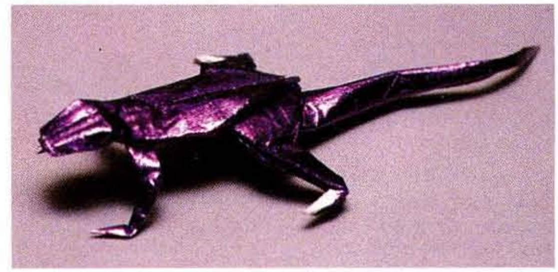


Figure 3: Crease pattern, base, and finished model of Max Hulme's "Lizard," a box- pleated design from a 4:1 rectangle. Below, Neal Elias's box-pleated masterpiece, "Llopio's Moment of Truth."

suited to rectangles. To design a new model, you may imagine yourself initially in possession of an infinitely long rectangle. You divide it up along the short side into twelfths or sixteenths. Each division is defined as one "unit." Beginning at one end of the rectangle, lay out the parts of the subject along the rectangle, allocating appropriate amounts of paper for each appendage (for example, a pair of points three units long requires six units of paper); when you've allocated all the parts, snip off the excess, and you have your starting rectangle. Crease on all the horizontal and vertical divisions; crease the diagonals; then starting from one end, collapse the paper on the pleats to form a base. Figure 3 shows the process for a lizard.

Some of the most fantastic structures origami has yet produced have resulted from box-pleating; a working Jack-in-the-box (several, actually), a steam locomotive, cars, trains, and planes. While the rectilinear lines of a box-pleated model are well-suited for man-made objects, and indeed, most box-pleated models are inanimate, there are a host of animals made using box-pleating, including one of the earliest: Elias's "Llopio's Moment of Truth," in which a bull, bullfighter, and cape are all folded from a single sheet (left). Box-pleated designs can get extremely involved. One of the most complex box-pleated models is my "Black Forest Cuckoo Clock" (shown on page 16); it is made from a 10:1 rectangle, contains about 200 meters of creases in a model 40 cm high, and takes 4 to 6 hours to fold.

Box-pleating brought complexity to rectangles, but the square remained inviolate until the

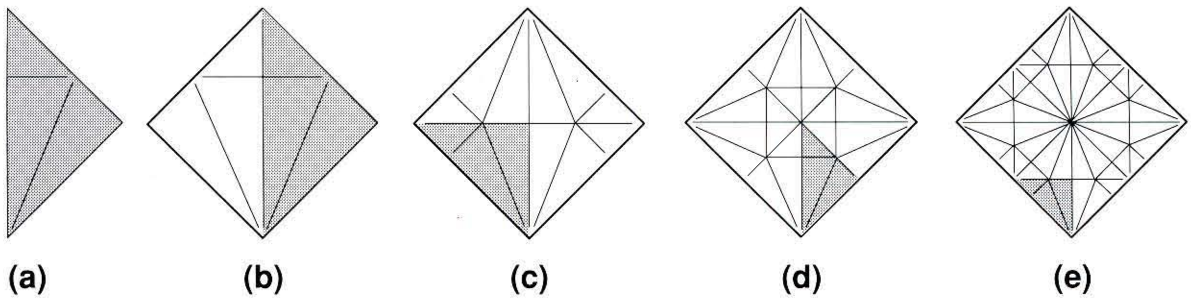


Figure 4: The basic shapes of technical folding. The triangle in (a) is found in the Kite Base (b), Fish Base (c), Bird Base (d), and Frog Base (e) in successively smaller sizes.

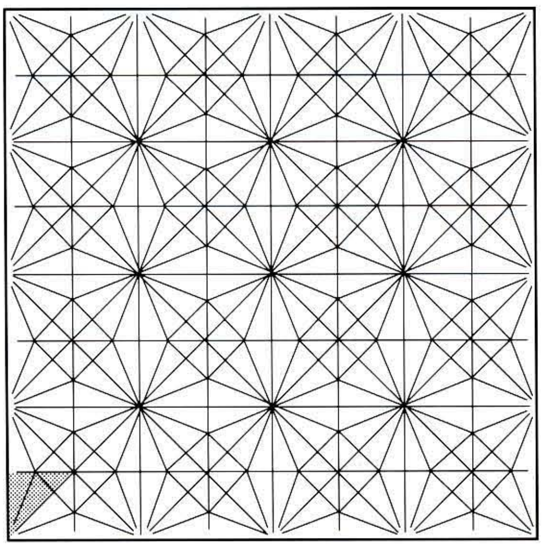


Figure 5: Crease pattern and model of Lang's "Sea Urchin," which contains 128 repetitions of the basic shape.

late 1970s and 1980s, when three Americans and a Japanese, working independently, hit upon a set of techniques and symmetries suitable for folding complex models from squares. They were John Montroll, a mathematician and computer scientist; Peter Engel, a science writer and architect; Jun Maekawa, a nuclear physicist; and myself. The techniques we developed have come to be called "technical" folding: origami composed of equal parts art and engineering.

The fundamentals of technical folding spring from the same geometric patterns present in the classic bases. The basic principle is quite simple. In the four classic bases, the same shape appears in multiples of two, four, eight, and sixteen. Technical folding simply expands upon that trend.

This reappearing shape is an isosceles right triangle with two creases in it; Figure 4 shows how it appears in each base in successively smaller sizes. Two of the basic triangles can be assembled into a square, yielding the Kite Base. Four give the Fish Base. Eight give the Bird Base. Sixteen give the Frog Base. The pattern is clear. We could easily go to thirty-two, in which case we would end up with a Blintzed Bird Base, the source of the previously mentioned elephant. As in box-pleated bases, the base may be formed from the creased square by collapsing the crease pattern on folds in alternate directions. Every source of radial creases becomes a point of the base. The more radial clusters of creases there are, the more appendages the final model may have. The crease pattern for my sea urchin (Figure 5), which incorporates 128 copies of this triangle, contains 25 equal-length points.

Figure 7: (right) Combining two A triangles and two B triangles gives a $1{:}\sqrt{2}$ rectangle. Four of these gives another $1{:}\sqrt{2}$ rectangle. Two of these, plus two Bird Bases gives the diagonal crease pattern shown, which is used in Engel's "Alligator," Montroll's "Shark," Maekawa's "Kangaroo," and Lang's "Triceratops."

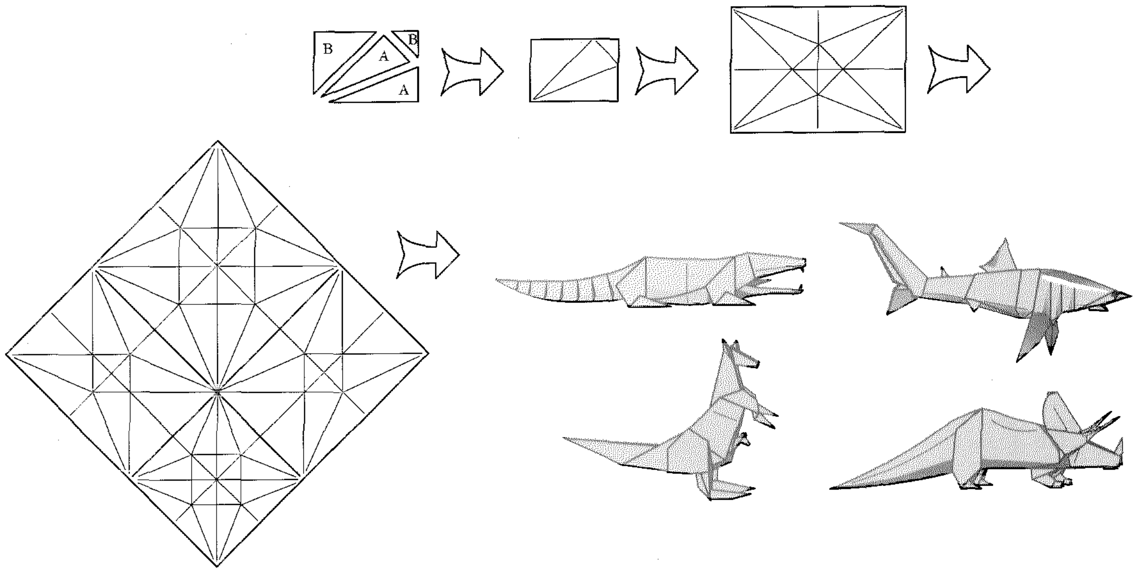
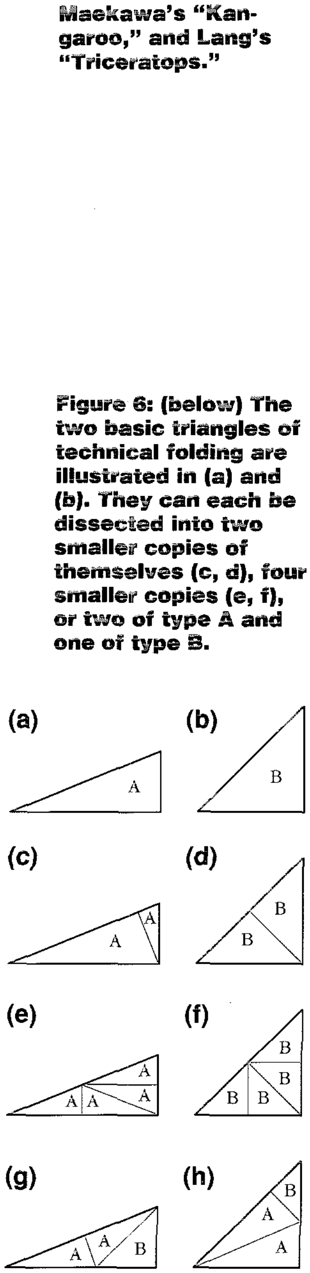


Figure 6: (below) The two basic triangles of technical folding are illustrated in (a) and (b). They can each be dissected into two smaller copies of themselves (c, d), four smaller copies (e, f), or two of type A and one of type B.



This triangle is not the most fundamental unit, however. It is composed of three smaller triangles: two identical scalene ($1{:}1{+}\sqrt{2}$) right triangles and one isosceles (1:1) right triangle that is a smaller copy of the original. We will call them type A and B triangles, respectively. These two triangles appear over and over in different sizes throughout the crease patterns in figures 4 and 5. They are the true building blocks of technical folding.

These two triangles have some interesting properties. They can each be dissected into two or four smaller copies of themselves, as shown in Figure 6(c-f). This property is not particularly unique, because any right triangle can be similarly dissected. What is interesting are the dissections shown in Figure 6(g-h); each triangle can be dissected into two triangles of type A and one of type B. By selectively applying these dissections to simple crease patterns, it is possible to get more complicated crease patterns, yielding more and more points.

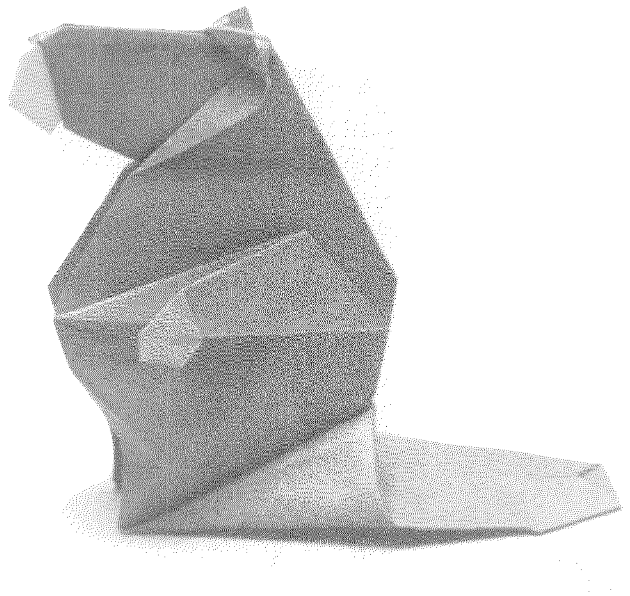
Alternatively, rather than breaking up a square into smaller and smaller triangles, we can assemble A and B triangles into larger and larger geometric patterns. By this means, we can create higher-order building blocks with which to generate bases. For example, two type As and two type Bs can be assembled into a $1{:}\sqrt{2}$ rectangle. Four of these rectangles can be assembled into another $1{:}\sqrt{2}$ rectangle with two axes of symmetry. Two of these rectangles can be combined with two Bird Bases to give the crease pattern in Figure 7, which yielded an alligator for Peter Engel, a shark for John Montroll, a kangaroo for Jun Maekawa, and a Triceratops for myself.

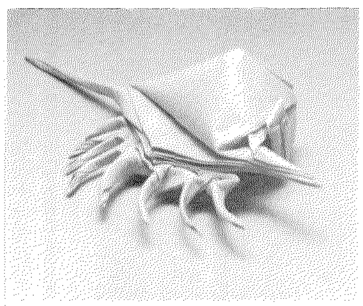
By combining ever larger assemblies of the basic modules, we can create ever more complicated bases, leading to ever more complex models. As we explore the different combinations of triangles, we can develop "libraries" of higher-order crease patterns; the $1{:}\sqrt{2}$ rectangle is an example of one. The problem of designing an origami base thereby reduces to that of tiling a square with A and B triangles (or higher-order combinations of such) so that we get a radial pattern of creases for each appendage of the model.

The set of patterns possible with this set of triangles is fundamentally richer than the set possible with box pleating. The reason is that we have two basically different shapes—types A and B—with which to construct initial crease patterns. All box pleating crease patterns, by contrast, can be produced using a single shape: the type B triangle. Two shapes give more possibilities for tiling and, therefore, more possible designs than one. The patterns possible with these two basic shapes are a rich trove of origami designs that is only beginning to be discovered.

As the technology to design origami models has improved, there has been a shift in the subjects that are folded toward the more challenging end of the spectrum. As is often the case in the sciences, we find technology in search of a problem to solve. The ability to fold multipointed creations cries out for a multipointed subject. Insects, once considered all but impossible, are now commonplace. Legs are no longer a sign of a realistic arthropod; mandibles are. The ulti-

With care, you should end up with your very own Caltech beaver.





Lang's "Murex."

mate challenge to a designer was once thought to be a lobster, with its eight long, skinny legs, two split claws, antennae and segmented tail. In 1970, lobsters didn't exist. In 1988, there are recognizable species.

Origami as art and as science

Science, via geometry and tesselation, has brought origami into the modern age. We may ask, what can origami bring to science? Origami has always enjoyed an interest among recreational mathematicians, and achieved a prominent appearance in Martin Gardner's "Mathematical Games" column in *Scientific American* magazine in 1960. It made it to the world of engineering in 1969, when Jon Myers, a scientist at Hughes Research Labs, published an article showing how origami could be used to simulate optical systems. Computing succumbed to the appeal of folded paper when, in 1971, Arthur Appel programmed an IBM System 360 computer to print out simple geometric configurations at the rate of more than a hundred a minute. Ninety percent were considered unsuccessful, but it raises an interesting question: could a computer someday design a model deemed superior to that designed by man? Since so much of the process of design is geometric, the prospect is not as outrageous as it may seem. Still, technical folding can only take us so far. The architects of technical folding have begun to lay the groundwork for systematic design. To our great surprise, origami, the ancient Eastern art, may be a science after all.

Accompanying this article are a piece of paper and instructions for a simple origami model. Carefully cut the square out on the black lines and follow the attached step-by-step directions. The instructions are written for someone with no prior experience in origami, but the following tips may help. Folds occur on dashed or dot-dot-dash lines. If the line is dashed, fold the paper toward you; if it is a dot-dot-dash line, fold it away from you. As you work through each step, look at the drawing; read the text; look ahead to the next drawing to see what the result should be; then fold as directed. With care, you should end up with your very own Caltech beaver. □

Robert Lang, an internationally known expert on origami, has been folding paper figures since the age of six. His book, The Complete Book of Origami *(Dover Publications) is scheduled to appear in early January, and another work,* Origami Zoo *(co-authored with Stephen Weiss), is in progress. A Caltech alumnus, Lang received his BS in electrical engineering in 1982 and PhD in applied physics in 1986 (with an MS from Stanford in between). He's currently employed at the Jet Propulsion Laboratory in the Photonics Group, Advanced Electronic Materials and Devices Section, where he works with lasers, not paper.*